1. *On Polarizations of Certain Homogeneous Spaces*

By Hideki OZEKI*) and Minoru WAKIMOTO**)

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1. It is one of main problems in the theory of unitary representations to find a unified way of constructing all irreducible unitary representations for an arbitrary Lie group.

Kostant ([7], [8]) has shown a very general method of constructing unitary representations, using polarizations of homogeneous sympletic spaces. And Kirillov ([5]) gave several problems related with Kostant's works.

In this note, we announce some results on invariant polarizations of homogeneous spaces by semi-simple Lie groups. First, an infinitesimal characterization of polarizations is given in Theorem 1. And examples in the third section would answer to one of the problems raised by Kirillov. Then the notion of principal nilpotent elements in a real semi-simple Lie algebra is introduced. Using this notion, we can show the existence of invariant polarizations of certain homogeneous spaces (Theorems 3 and 4). An exposition with detailed proofs will be published elsewhere.

2. Let G be a connected Lie group, $\mathfrak{g}_0$ its Lie algebra and $\mathfrak{g}_0'$ the dual vector space of $\mathfrak{g}_0$. The space $\mathfrak{g}_0'$ has the G-module structure contragredient to the adjoint representation of G on $\mathfrak{g}_0$. For an element f of $\mathfrak{g}_0$, we denote by G^f the isotropy subgroup of G with respect to f, and by $\mathfrak{g}_0^f$ the subalgebra of $\mathfrak{g}_0$ corresponding to G^f. Kostant has shown that every G-orbit $G(f)=G/G^f$ has a canonical invariant symplectic structure.

Let $\mathfrak{g}$ and $\mathfrak{g}^f$ be the complexification of $\mathfrak{g}_0$ and $\mathfrak{g}_0^f$. For a complex subalgebra $\mathfrak{p}$ of $\mathfrak{g}$, we consider the following conditions:

i) $\mathfrak{g}^f \subset \mathfrak{p}$;

ii) $f([\mathfrak{p}, \mathfrak{p}])=\{0\}$;

iii) $\dim \mathfrak{p}-\dim \mathfrak{g}^f=\dim \mathfrak{g}-\dim \mathfrak{p}$;

iv) $\mathfrak{p}$ is $Ad(G^f)$-stable;

v) $\mathfrak{p}+\sigma(\mathfrak{p})$ is a complex subalgebra of $\mathfrak{g}$, where σ denotes the conjugation of $\mathfrak{g}$ with respect to $\mathfrak{g}_0$.

Definition. For f in $\mathfrak{g}_0'$ and a complex subalgebra $\mathfrak{p}$ of $\mathfrak{g}$, $\mathfrak{p}$ is called

1) *a weak polarization* of f if $\mathfrak{p}$ satisfies i)—iii),

2) *a polarization* of f if $\mathfrak{p}$ satisfies i)—iv),

*) Department of Mathematics, Osaka University.

**) Department of Mathematics, Hiroshima University.