22. *On the Gaps between the Refinements of the Increasing Open Coverings*

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(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1972)

§1. Preliminaries. In normal spaces, C. H. Dowker [1] gave the characterizations of countably paracompact spaces. After a time, F. Isikawa [3] discussed about the generalizations of its characterizations.

One of our purposes, in this paper, is to study the gaps, in which their characterizations of countably paracompact spaces need not be identical without the normality. Recently, P. Zenor [7] (resp. S. Sasada [4]) defined the topological class which was contained in the countably paracompact class and was a generalization of the property being stated by F. Isikawa [3] (resp. C. H. Dowker [1]: Theorem 2).

Another purpose of this paper is to find the gaps between the above topological spaces. Before stating properties, we will recall or define the terms which are used in this paper.

Let $\mathfrak{A}=\{A_\alpha \mid \alpha \in A\}$ be a collection of subsets of a topological space X. $\mathfrak{A}$ is said to be *monotone increasing* (resp. *monotone decreasing*) if A is well ordered and $A_\alpha \supseteq A_\beta$ (resp. $A_\alpha \subseteq A_\beta$) for each α, $\beta \in A$ with $\alpha \geqslant \beta$. The space X is said to have a *$\mathfrak{B}$-property* (resp. *a weak $\mathfrak{B}$-property*) if for each monotone decreasing family $\{F_\alpha / \alpha \in A\}$ of closed sets of X with vacuous intersection there is a monotone decreasing family (resp. a simple collection) $\{G_\alpha \mid \alpha \in A\}$ of open sets of X such that $\bigcap_{\alpha \in A} \overline{G}_\alpha$[1] $=\emptyset$ and $G_\alpha \supset F_\alpha$ for each $\alpha \in A$. X is said to have a *countable $\mathfrak{B}$-property* (resp. a *countable weak $\mathfrak{B}$-property*) if the indexed set A of the definition of the $\mathfrak{B}$-property (resp. the weak $\mathfrak{B}$-property) is the countable set. X is a *$\mathfrak{B}$-space* (resp. *a weak $\mathfrak{B}$-space, a countable $\mathfrak{B}$-space, a countable weak $\mathfrak{B}$-space*) if X has the $\mathfrak{A}$-property (resp. the weak $\mathfrak{B}$-property, the countable $\mathfrak{B}$-property, the countable weak $\mathfrak{B}$-property).

There is natural dual characterizations of $\mathfrak{B}$-property etc. in terms of the monotone increasing open covering (see T. Tani and Y. Yasui [6]). The following properties are equivalent: (1) the countable $\mathfrak{B}$-property (2) the countable weak $\mathfrak{B}$-property, (3) the countable paracompactness (see F. Ishikawa [3]).

The normal $\mathfrak{B}$-space (resp. the normal weak $\mathfrak{B}$-space) is said to be

1) $\overline{G}_\alpha$ denotes the closure of G_α.