42. L-ideals of Measure Algebras

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1. Introduction. Let G be a non-discrete locally comapct abelian group with the dual group Γ of G. We will denote by $M(G)$ the Banach algebra of all bounded regular Borel measures on G under convolution multiplication. If $\mu, \nu \in M(G)$, then their convolution product will be denoted $\mu * \nu$. We shall use additive notation for the group operation in G.

If $\mu, \nu \in M(G)$, then "$\nu \ll \mu$" will mean "ν is absolutely continuous with respect to μ" and "$\mu \perp \nu$" will mean "μ and ν are mutually singular". If $\mathfrak{M}$ is a closed subspace (subalgebra, ideal) of $M(G)$ will be called an *L-subspace* (*L-subalgebra*, *L-ideal*) provided $\mu \in \mathfrak{M}, \nu \in M(G)$ and $\nu \ll \mu$ imply $\nu \in \mathfrak{M}$. If $\mathfrak{M}$ is an L-subspace and $\mu \in M(G)$, then we say $\mu \perp \mathfrak{M}$ provided $\mu \perp \nu$ for each $\nu \in \mathfrak{M}$. We set $\mathfrak{M}^{\perp} = \{\mu \in M(G) : \mu \perp \mathfrak{M}\}$.

It is known that *there exists a compact commutative topological semigroup S with identity and an order preserving isometric isomorphism θ of* $M(G)$ into $M(S)$ such that:

T-(a) *the image of $M(G)$ in $M(S)$ is weak-$*$ dense*:

T-(b) *each multiplicative linear functional h on $M(G)$ has the form $h(\mu) = \int_S f d\theta\mu$ for some non-zero continuous semicharacter on S*;

T-(c) *there are enough non-zero continuous semicharacter on S to separate points*; and

T-(d) *if $\mu \in M(G)$, $\nu \in M(S)$ and $\nu \ll \theta\mu$ then there is a measure $\omega \in M(G)$ such that $\omega \ll \mu$ and $\theta\omega = \nu$* (cf. [2]).

We call S the *structure semigroup* of $M(G)$. The space of all non-zero continuous semicharacters on S is denoted by $\hat{S}$. We may consider $\hat{S}$ to be the maximal ideal space of $M(G)$, if we define the Gelfand transform of $\mu \in M(G)$ by $\hat{\mu}(f) = \int_S f d\theta\mu$ for $f \in \hat{S}$, and give $\hat{S}$ the weakest topology under which all of the functions $\hat{\mu}$ for $\mu \in M(G)$ are continuous. Since $M(G)$ has identity, $\hat{S}$ is a compact semigroup under pointwise multiplication. Pointwise multiplication is not generally continuous in the Gelfand topology. However, for fixed $g \in \hat{S}$ it is easily seen that the map $f \to gf$ is weakly continuous. We may consider Γ to be the maximal group at identity. In other word, $\Gamma = \{f \in S : |f| \equiv 1\}$. As well known, if $\mu \in M(G)$ and $\hat{\mu}(f) = 0$ for all $f \in \Gamma$, then $\mu = 0$.