72. On Two Classes of Subalgebras of $L^1(G)$

By Leonard Y. H. YAP
Department of Mathematics University of Singapore

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1. Introduction. Let G and $\hat{G}$ be two locally compact Abelian groups in Pontrjagin duality. The Fourier transform of a function $f \in L^1(G)$ will be denoted by $\hat{f}$. For $1 \leqq p < \infty$, define

$$A^p(G)=\{f \in L^1(G) : \hat{f} \in L^p(\hat{G})\}, \qquad B^p(G)=L^1(G) \cap L^p(G).$$

The space $A^p(G)$ is a Banach algebra with respect to the norm $\|\cdot\|_{A^p(G)}$ defined by $\|f\|_{A^p(G)}=\|f\|_1+\|\hat{f}\|_p$ and the usual convolution product. The Banach algebra $A^p(G)$ have been studied by Larsen-Liu-Wang [8], Lai [5]–[7], Martin-Yap [9], and others. The space $B^p(G)$ is a Banach algebra with respect to the norm $\|\cdot\|_{B^p(G)}$ defined by $\|f\|_{B^p(G)}=\|f\|_1+\|f\|_p$ and the usual convolution product. The Banach algebras $B^p(G)$ have been studied by Warner [12], Yap [15], and others. The purpose of this paper is to extend some of the results on $A^p(G)$ and $B^p(G)$ to the spaces

$$A(p, q)(G)=\{f \in L^1(G) : \hat{f} \in L(p, q)(\hat{G})\}$$

and

$$B(p, q)(G)=L^1(G) \cap L(p, q)(G)$$

respectively (see next section for the definition of $L(p, q)(G)$ and some relevant facts about these spaces). In Section 2 we identify the maximal ideal spaces of the algebras $A(p, q)(G)$ and $B(p, q)(G)$, show that they satisfy Ditkin's condition and that the Shilov-Wiener Tauberian theorem holds for these algebras. In Section 3 we prove non-factorization theorems for these algebras.

2. Tauberian theorem for $A(p, q)(G)$ and $B(p, q)(G)$. For the convenience of the reader, we now review briefly what we need from the theory of $L(p, q)$ spaces.

Definition 2.1. Let f be a measurable function defined on (G, λ), where λ is the Haar measure of G. For $y \geq 0$, we define

$$m(f, y)=\lambda\{x \in G : |f(x)|>y\}.$$

For $x \geq 0$, we define

$$\begin{aligned} f^*(x) &= \inf\{y : y>0 \text{ and } m(f, y) \leqslant x\} \\ &= \sup\{y : y>0 \text{ and } m(f, y)>x\}, \end{aligned}$$

with the conventions $\inf \phi=\infty$ and $\sup \phi=0$. For $x>0$, we define

$$f^{**}(x)=x^{-1}\int_0^x f^*(t)dt.$$

We also define