# 110. *On the Propagation of Error in Numerical Integrations*

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**§0. Introduction.** Even with quite simple differential equations, it can happen that their solutions are not expressible in a closed form and that a numerical approach is the most convenient way to deal with the problem. And in this case if an approximate value $y_n$ of the solution $y(x)$ of a differential equation at the point $x_n$ has been calculated by some approximate methods, the estimate on the magnitude of error

$$e_n = y_n - y(x_n) \qquad (n=1,2,3,\cdots) \tag{0.1}$$

is of great importance.

While we possess simple and useful error estimate for the propagation of error, it seems, however, that if we concern with the problem of asymptotic behavior of the propagation of error, not so many results appeared. The purpose of this paper is to state some results on a propagation of error of some approximate equations.

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**§1.** First we consider the first order differential equation:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0. \end{cases} \tag{1.1}$$

We shall now try to approximate the equation (1.1) by the difference equation:

$$y_{n+1} = y_n + hf(x_n, y_n) \tag{1.2}$$

which is known as Euler's method.

In actual calculation, the calculated value of $y_{n+1}$ is given by the formula:

$$y_{n+1} = y_n + hf(x_n, y_n) - R_{n+1} \qquad (R_n:\ \text{round-off error}) \tag{1.3}$$

On the other hand, if we denote the true value of the solution of (1.1) at the point $x = x_n$ by $y(x_n)$, we have also the relation:

$$y(x_{n+1}) = y(x_n) + hf(x_n, y(x_n)) + T_{n+1}, \tag{1.4}$$

where $T_n$ denotes the truncation error corresponding to the $n$-th step. If we subtract (1.3) from (1.4) and write

$$E_n = T_n + R_n, \tag{1.5}$$

we find the difference equation:

$$e_{n+1} = e_n + h(f(x_n, y(x_n)) - f(x_n, y_n)) + E_{n+1}. \tag{1.6}$$