# 150. *On Closed Graph Theorem*

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The closed graph theorem has been proved in some linear topological space. In this note we show that this theorem is true in a ranked space with some conditions. The theory of ranked space has been investigated by K. Kunugi since 1954. Throughout this note, $g, f, \cdots$ will denote points of a ranked space, $U_i, V_i, \cdots$ neighbourhoods of the origin with rank $i$, $\{U_{\gamma_i}\}, \{V_{\gamma_i}\}, \cdots$ fundamental sequences of neighbourhoods with respect to the origin. Let a linear space $E$ be a complete ranked space with indicater $\omega_0$, which satisfies the following conditions.

(E, 1) (1) For any neighbourhood $U_i$, the origin belongs to $U_i$.
(2) The $E$ is the neighbourhood of the origin with rank zero.

(E, 2) Let $U_i$ be any neighbourhood of the origin, $\lambda$ be any number with $\lambda > 0$ and $g$ be a point in $\lambda U_i$. If $\{V_{\gamma_i}\}$ is a fundamental sequence of neighbourhoods, there is an integer $i_0$ such that $\lambda U_i \supset g + V_{\gamma_j}$ for $j \geqq i_0$.

(E, 3) The following conditions are the modification of Washihara's conditions [4].

(R, $L_1$) For any $\{U_i\}$ and $\{V_i\}$, there is a $\{W_i\}$ such that $U_i + V_i \subseteqq W_i$.

(R, $L_2$)′ (1) For any $\{U_i\}$ and $\lambda > 0$, there is a $\{V_i\}$ such that $\lambda U_i \subseteqq V_i$.
(2) For any $\{U_i\}$ and $\{\lambda_i\}$ with $\lim \lambda_i = 0$, $\lambda_i > 0$, there is a $\{V_i\}$ such that $\lambda_i U_i \subseteqq V_i$.

(R, $L_3$) Let $g$ be any point in $E$. For any $\{U_i\}$ there is a $\{V_i(g)\}$, which is a fundamental sequence of neighbourhoods with respect to $g$, such that $g + U_i \subseteqq V_i(g)$ and conversely, for any $\{U_i(g)\}$ there is a $\{V_i\}$ such that $U_i(g) \subseteqq g + V_i$.

(E, 4) Let $M$ be an absolutely convex set in $E$ and $V_i$ be a neighbourhood of the origin. If $\bar{M}$[1] $\supset f + V_i$, there is a $\lambda > 0$ such that $\bar{M} \supset \lambda V_i$.

(E, 5) For given distinct points $g_1, g_2$, there exists some neighbourhood of the origin $U_i$ such that $(g_1 + U_i) \not\ni g_2$.

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1) $g \in \bar{M}$ if and only if there exists some sequence $\{g_i\}$ in $M$ such that $g_i \to g$ in the sense of ranked space.