146. On the Structure of Fourier Hyperfunctions*)

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We show below a complete analogue of the following structure theorem for the temperate distributions: Every element $u \in \mathcal{S}'$ can be expressed in the form $u=(1-\Delta)^N f$, where f is a temperate continuous function. Thus Corollary 1.13 in [3] is improved, and Remark 1.15 there should be cut away. We refer to [3] for the terminology employed here.

Theorem. *For every Fourier hyperfunction $u \in Q$ we can find an elliptic local operator $J(D)$ and an infinitely differentiable function $f(x)$ of infra-exponential growth satisfying $u=J(D)f$.*

By the word "infra-exponential" we mean the following type of estimate:

$$|f(x)| \leq C_\varepsilon \exp(\varepsilon|x|), \quad {}^\forall\varepsilon>0, \quad {}^\exists C_\varepsilon>0.$$

Note that a continuous function of infra-exponential growth is "temperate" in the sense of hyperfunction theory. Especially it can be considered as a Fourier hyperfunction in a standard way.

Now let us say that a continuous function $\psi(r) \geq 0$ of one variable $r \geq 0$ is infra-linear if it satisfies the estimate

$$\psi(r) \leq \varepsilon r + C_\varepsilon, \quad {}^\forall\varepsilon>0, \quad {}^\exists C_\varepsilon>0.$$

Before the proof of our theorem we prepare

Lemma. *Let $\psi_k(r)$, $k=1,2,\cdots$, be a sequence of infra-linear functions. Then we can find an infra-linear function $\psi(r)$ and a sequence of constants C_k, $k=1,2,\cdots$, satisfying*

$$\psi_k(r) \leq \psi(r) + C_k. \tag{1}$$

Proof. Approximating the graphs of $\psi_k(r)$ by polygons from above, and smoothing the corners, we can assume that $\psi_k(r)$ are monotone increasing, concave and differentiable. Further, replacing $\psi_k(r)$ by $\sum_{j=1}^k \psi_j(r)$ if necessary, we can assume that $\psi_k(r) \leq \psi_l(r)$ and $\psi'_k(r) \leq \psi'_l(r)$ for $k \leq l$.

Now choose a_k by the following induction process:

$$\psi'_k(a_k) \leq \frac{1}{k}, \tag{2}$$

$$\frac{\psi_k(a_k)-\psi_k(a_{k-1})}{a_k-a_{k-1}} \leq \frac{1}{k}. \tag{3}$$

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