162. *Countable Structures for Uncountable Infinitary Languages*

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In model theory of infinitary languages with countable conjunctions and finite strings of quantifiers in the sense of H. J. Keisler's book [3], we have some theorems which hold even in the case that there are uncountably many non-logical symbols, e.g. countable isomorphism theorem and countable definability theorem (cf. Scott [4], Chang [1] and Kueker [2]). Of course we have theorems which hold only in the case that there are at most countably many non-logical symbols, e.g. the existence theorem of Scott's sentence (cf. [3]).

In order to make clear the distinction between two kinds of theorems above mentioned we shall show that for each countable structure $\mathfrak{A}$, which is associated to an uncountable infinitary language L, there is a countable sublanguage L_0 of L such that every formula in L is definable in $\mathfrak{A}$ by a formula in L_0. We use the standard model theoretic terminology (cf. [2] and [3]). Let L be a first order language with countable conjunctions and finite strings of quantifiers and possibly uncountably many non-logical symbols. Then we have the following

Theorem. *Let $\mathfrak{A}$ be a countable structure for L. Then there is a countable sublanguage L_0 of L such that for each formula $\varphi(v_1, v_2, \cdots, v_n)$ in L there is a formula $\psi(v_1, v_2, \cdots, v_n)$ in L_0 such that*

$$\mathfrak{A} \models (\forall v_1)(\forall v_2)\cdots(\forall v_n)(\varphi(v_1, v_2, \cdots, v_n) \leftrightarrow \psi(v_1, v_2, \cdots, v_n)).$$

Proof. For each sequence $\sigma = \langle L', a_1, \cdots, a_n \rangle$, where L' a countable sublanguage of L and $a_1, \cdots, a_n$ are elements of $|\mathfrak{A}|$, let φ_σ be the Scott's sentence of the structure $(\mathfrak{A} \upharpoonright L', a_1, \cdots, a_n)$ which is obtained from $\mathfrak{A} \upharpoonright L'$, the reduct of $\mathfrak{A}$ to L', by adding $a_1, \cdots, a_n$ as new individuals. Then there is a formula $\varphi_\sigma(v_1, \cdots, v_n)$ in L' such that $\varphi_\sigma = \varphi_\sigma(a_1, \cdots, a_n)$, i.e. the sentence φ_σ is obtained from the formula $\varphi_\sigma(v_1, \cdots, v_n)$ by replacing $v_1, \cdots, v_n$ by $a_1, \cdots, a_n$ respectively. (We identify the elements a_i in $|\mathfrak{A}|$ and the constant symbols a_i corresponding to them.) Then for each $b_1, \cdots, b_n$ in $|\mathfrak{A}|$, we have

(1) $\mathfrak{A} \models \varphi_\sigma[b_1, \cdots, b_n] \Leftrightarrow (\mathfrak{A} \upharpoonright L', a_1, \cdots, a_n) \cong (\mathfrak{A} \upharpoonright L', b_1, \cdots, b_n).$

Hence if $\sigma_1 = \langle L_1, a_1, \cdots, a_n \rangle$, $\sigma_2 = \langle L_2, a_1, \cdots, a_n \rangle$ and $L_1 \subseteq L_2$, then we have

(2) $\mathfrak{A} \models (\forall v_1)\cdots(\forall v_n)(\varphi_{\sigma_2}(v_1, \cdots, v_n) \rightarrow \varphi_{\sigma_1}(v_1, \cdots, v_n)).$