57. An Application of a Certain Argument about Isomorphisms of α-Saturated Structures

By Hiroyoshi TABATA
Nara Technical College

(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1973)

Applying A. Robinson's proof of the Completeness Theorem, Y. Nakano [3] recently gave a proof of the theorem of Grätzer [2; p. 138, Theorem 6] on the existence of homomorphisms under certain conditions. But, in the theory of models, there is a well known argument about isomorphisms of α-saturated structures (cf. [1; Chap. 11]). As an application of this argument, we shall give a simplified proof of an extended version of the Grätzer's theorem.

We consider each ordinal number as coinciding with the set of smaller ordinal numbers. We use letters ξ, ζ, ρ to denote ordinal numbers and n, i, k to denote natural numbers. We regard cardinals as being identical with initial ordinals. If X is a set we denote its cardinal by $\overline{\overline{X}}$.

Let ρ be an arbitrary ordinal number and let μ be a sequence of natural numbers with domain ρ ($\mu \in \omega^\rho$). By a relational structure of type μ we shall mean a sequence $\mathfrak{A}=\langle A, R_\xi^{\mathfrak{A}}\rangle_{\xi\in\rho}$ where A, the domain of $\mathfrak{A}$, is a non-empty set and $R_\xi^{\mathfrak{A}}$ is a $\mu(\xi)$-ary relation on A for each $\xi<\rho$. Throughout our discussion we shall assume that $\mu\in\omega^\rho$ is some fixed type, that all relational structures we mention are of this type, that L is the appropriate first order language for structures of this type and that for each ordinal ξ, L_ξ is the language obtained from L by adding the ξ-termed sequence of new and distinct constants $\langle c_\zeta : \zeta\in\xi\rangle$. For any relational structure $\mathfrak{A}=\langle A, R_\xi^{\mathfrak{A}}\rangle_{\xi\in\rho}$ and for any ξ-termed sequence $\vec{a}=\langle a_\zeta : \zeta\in\xi\rangle$ of elements of A, we use $(\mathfrak{A}, \vec{a})$ to denote the structure for L_ξ obtained from $\mathfrak{A}$ by interpreting each c_ζ by a_ζ.

Satisfaction of formulas of L_ξ in a structure for L_ξ is defined as usual. If θ is a formula whose free variables are among $v_0, \cdots, v_n$ and if θ holds in $\mathfrak{A}$ with respect to the elements $e_0, \cdots, e_n$ of the domain of $\mathfrak{A}$, then we write $\mathfrak{A}\models\theta[e_0, \cdots, e_n]$.

We use $F(L)$ to designate the set of all formulas of L having at most the one variable v_0 free, and we use $F(L_\xi)$ to designate the corresponding set of formulas of L_ξ.

Suppose Σ is a set of formulas from $F(L_\xi)$, $\mathfrak{A}$ is a relational structure for L and $\vec{a}\in A^\xi$. We say that Σ is simultaneously satisfiable in