99. On Expandability

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In [1] Katětov proved the following useful theorem:

A normal space X is collectionwise normal and countably paracompact if and only if

(*) for every locally finite collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X there exists a locally finite collection $\{G_\lambda \mid \lambda \in \Lambda\}$ of open subsets of X such that $F_\lambda \subset G_\lambda$ for every $\lambda \in \Lambda$.

Recently, Krajewski [3] has called a topological space X *expandable* if X satisfies this condition (*). Smith and Krajewski [4] have introduced some generalizations (almost expandability, etc.) of expandability, and they have obtained various results concerning these notions.

In this paper, we shall introduce new notions of θ-expandability, subexpandability etc., and obtain analogous results. Furthermore, we shall study additional properties of expandable spaces, θ-expandable spaces etc.

The proofs and details of the results will be published elsewhere.

1. A collection $\mathfrak{A}$ of subsets of a space X is said to be *bounded locally finite* [2], if there exists a positive integer n such that every point of X has a neighborhood which intersects at most n elements of $\mathfrak{A}$. Every discrete collection is bounded locally finite and every bounded locally finite collection is locally finite.

A space X is said to be *θ-expandable* (resp. *boundedly θ-expandable* or *discretely θ-expandable*), if for every locally finite (resp. bounded locally finite or discrete) collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X there exists a sequence $\mathfrak{G}_n = \{G_{\lambda,n} \mid \lambda \in \Lambda\}$, $n = 1, 2, \cdots$, of collections of open subsets of X satisfying the following two conditions:

(1) $F_\lambda \subset G_{\lambda,n}$ for each $\lambda \in \Lambda$ and each n.

(2) For each point x of X there exists a positive integer n such that only finitely many elements of $\mathfrak{G}_n$ contain x.

Theorem 1.1. (a) *X is boundedly θ-expandable if and only if X is discretely θ-expandable.*

(b) *X is θ-expandable if and only if X is discretely θ-expandable and countably θ-refinable.*

(c) *A θ-refinable space is θ-expandable.*

A space X is said to be *discretely subexpandable*, if for every discrete collection $\{F_\lambda \mid \lambda \in \Lambda\}$ of subsets of X there exists a sequence