88. *An Example of Temporally Inhomogeneous Scattering*

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§ 1. The result. Consider a system of linear partial differential equations

$$\frac{\partial u(x,t)}{\partial t}=\sum_{j=1}^{n} A_j(x,t)\frac{\partial u(x,t)}{\partial x_j}+B(x,t)u(x,t). \tag{1.1}$$

Here $u=(u_1,\cdots,u_N)$ is an N-vector of unknown functions of x and t; $A_j(x,t)$ and $B(x,t)$ are $N\times N$ matrix functions, and $A_j(x,t)$ are assumed to be Hermitian symmetric.

In order to guarantee the existence and the uniqueness of the solution $u(x,t)\in\mathcal{E}_t^1(L^2(\boldsymbol{R}^n))\cap\mathcal{E}_t^0(H^1(\boldsymbol{R}^n))$[1] of (1.1) with Cauchy data $u(x,0)=u_0(x)\in H^1(\boldsymbol{R}^n)$, we assume the following (see [5], [6]):

(I) (a) The maps $t\mapsto A_j(\cdot,t)$ are continuous on $(-\infty,\infty)$ to $\mathcal{B}^1(\boldsymbol{R}^n)$,

(b) $t\to B(\cdot,t)$ is continuous on $(-\infty,\infty)$ to $\mathcal{B}^0(\boldsymbol{R}^n)$ and

$$\frac{\partial B(x,t)}{\partial x_j}\in\mathcal{B}^0(\boldsymbol{R}^n\times(-\infty,\infty)),\qquad j=1,2,\cdots,n.$$

Here $\mathcal{B}^l(\boldsymbol{R}^m)$ is the set of all $N\times N$-matrix valued functions A such that A and $D^\alpha A$, $|\alpha|\leqq l$ are continuous and bounded on $\boldsymbol{R}^m$.

We further consider two systems of linear partial differential equations given by

$$\frac{\partial u^\pm(x,t)}{\partial t}=\sum_{j=1}^{n} A_j^\pm\frac{\partial u^\pm(x,t)}{\partial x_j}+B^\pm u^\pm(x,t) \tag{1.2$^\pm$}$$

where $A_j^\pm$ are $N\times N$ constant Hermitian symmetric matrices and $B^\pm$ are $N\times N$ constant matrices satisfying $B^\pm+(B^\pm)^*=0$. (F^* denotes the Hermitian conjugate matrix of F.)

We assume that $(1.2)^\pm$ are close to (1.1) near $|t|=\infty$ in the following sense.

(II) There exists a function $\phi(t)\in L^1(-\infty,\infty)$ satisfying

$$|A_j(x,t)-A_j^\pm|_{\mathcal{B}^1(\boldsymbol{R}^n)}\leqq\phi(t),\quad |B(x,t)-B^\pm|_{\mathcal{B}^1(\boldsymbol{R}^n)}\leqq\phi(t)\qquad\text{for } t\gtrless 0. \tag{1.3}$$

We define an operator $U(t;s)$ by $U(t;s)u_0=u(x,t)$ where $u(x,t)\in\mathcal{E}_t^1(L^2(\boldsymbol{R}^n))\cap\mathcal{E}_t^0(H^1(\boldsymbol{R}^n))$ is a solution of (1.1) with Cauchy data $u_0(x)\in H^1(\boldsymbol{R}^n)$ at time s. We define the operators $U_0^\pm(t;s)$ analogously. By the energy inequality, expressed in Lemma 1 and Lemma 2 below, the

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1) $u(x,t)\in\mathcal{E}_t^l(H^k(\boldsymbol{R}^n))$ means that $u(\cdot,t)$ is a $H^k(\boldsymbol{R}^n)$ valued function of t, l-times continuously differentiable with respect to t in $H^k(\boldsymbol{R}^n)$-norm.