86. *Oscillatory Integrals of Symbols of Pseudo-Differential Operators on R^n and Operators of Fredholm Type*

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Introduction. In this paper we shall introduce the oscillatory integral of the form $O_s-\iint e^{-ix\cdot\xi}p(\xi,x)dxd\xi$ for a C^∞-function $p(\xi,x)$ of class $\mathcal{A}$ (defined in Section 1), and by using this integral study the algebra of pseudo-differential operators of class $S^m_{\lambda,\rho,\delta}$, $0\leqq\delta\leqq\rho\leqq 1, \delta<1$, whose basic weight function $\lambda=\lambda(x,\xi)$ varies even in x and may increase in polynomial order.*) The Friedrichs part P_F of the operator P of class $S^m_{\lambda,\rho,\delta}$ will be defined as in Kumano-go [6]. Then, the L^2-boundedness for the operator P of class $S^0_{\lambda,\rho,\delta}$ for $\delta<\rho$, can be proved by using P_F and the Calderon-Vaillancourt theorem in [1]. We have to note that all the results obtained there hold even for operator-valued symbols as in Grushin [3].

Next we shall give a sufficient condition in order that an operator of class $S^m_{\lambda,\rho,\delta}$ is Fredholm type. Finally we shall derive a similar inequality to that of Grushin [3] for an operator with polynomial coefficients and with mixed homogeneity in (x,ξ), and give a theorem on hypoellipticity at the origin.

All the theorems are stated without proofs and the detailed description will be published elsewhere.

§ 1. Oscillatory integrals.

Definition 1.1. We say that a C^∞-function $p(\xi,x)$ in $R^{2n}_{\xi,x}$ belongs to a class $\mathcal{A}^m_\delta$, $-\infty<m<\infty, 0\leqq\delta<1$, when for any multi-index α,β we have

$$|p^{(\alpha)}_{(\beta)}(\xi,x)|\leqq C_{\alpha,\beta}\langle x\rangle^{l_\beta}\langle\xi\rangle^{m+\delta|\beta|} \tag{1.1}$$

for constants $C_{\alpha,\beta}$ and l_β, where $p^{(\alpha)}_{(\beta)}=\partial^\alpha_\xi D^\beta_x p$, $D_{x_j}=-i\partial/\partial x_j$, $\partial_{\xi_j}=\partial/\partial\xi_j$, $j=1,\cdots,n$, $\langle x\rangle=\sqrt{1+|x|^2}$, $\langle\xi\rangle=\sqrt{1+|\xi|^2}$. We set

$$\mathcal{A}=\bigcup_{0\leqq\delta<1}\ \bigcup_{-\infty<m<\infty}\mathcal{A}^m_\delta$$

(cf. [8]).

Definition 1.2. For a $p(\xi,x)\in\mathcal{A}$ we define the oscillatory integral $O_s[p]$ by

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*) R. Beals and C. Fefferman have reported to us that they discovered a new class $S^{M,m}_{\Phi,\phi}$ of pseudo-differential operators, which is defined by basic weight functions $\Phi(x,\xi)$ and $\phi(x,\xi)$ depending on x and ξ, and covers Hörmander's class $S^m_{\rho,\delta}$ in [4].