130. On Some Examples of Non-normal Operators. IV

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(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 12, 1973)

1. **Introduction.** Throughout the note, we shall consider a (bounded linear) operator T acting on a Hilbert space $\mathfrak{H}$ with the spectrum $\sigma(T)$ and the numerical range $W(T)$. Let us denote further that

(1) $$r(T)=\sup\{|\lambda|;\lambda\in\sigma(T)\}$$

and

(2) $$w(T)=\sup\{|\lambda|;\sigma\in W(T)\}.$$

An operator T is called a *normaloid* if $\|T\|=r(T)$ and a *spectraloid* if $r(T)=w(T)$. T is called a *transaloid* if T satisfies that $\|T-\lambda\|=r(T-\lambda)$ for any complex number λ. Clearly, a transaloid is a normaloid, and conversely T is a transaloid if and only if $T-\lambda$ is a normaloid for every λ. T is called a *convexoid* if $\overline{W}(T)=\mathrm{co}\,\sigma(T)$ where $\overline{W}(T)$ is the closure of $W(T)$ and $\mathrm{co}\,S$ is the convex hull of a set S in the complex plane. T is called to satisfy (G_1) if

(3) $$\|(T-\lambda)^{-1}\|\leqq\frac{1}{\mathrm{dist}\,(\lambda,\sigma(T))}$$

for $\lambda\notin\sigma(T)$. A transaloid is a convexoid, and an operator satisfying (G_1) is a convexoid by [4].

In the present note, we shall characterize transaloids in terms of spectral sets and dilations in §§2–3. In §4, we shall discuss some examples of non-normal operators to disprove certain conjectures which naturally arise from [2]. In this note, we shall denote conveniently by D the unit disk of the complex plane.

2. **Spectral sets.** A (closed) set S in the plane is a *spectral set* for an operator T if

(4) $$\sigma(T)\subset S$$

and

(5) $$\|f(T)\|\leqq\|f\|_S$$

for any rational function f with poles off S, where

$$\|f\|_S=\sup\{|f(\lambda)|;\lambda\in S\},$$

cf. [5] and [7]. The following theorem is fundamental:

Theorem A (von Neumann [7]). *$\{\lambda;|\lambda-\mu|\leqq k\}$ is a spectral set for an operator T if and only if $\|T-\mu\|\leqq k$.*

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