153. On Exceptional Linear Combinations of Entire Functions

By Masakimi KATO
Department for Liberal Art, Shizuoka University

(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 12, 1973)

1. Introduction.

As an interesting result with respect to the relations between the sum of deficiencies and the number of Picard's exceptional values of entire algebroid functions, K. Niino and M. Ozawa [3], M. Ozawa [5] and T. Suzuki [6] showed the following fact: Let $f(z)$ be a transcendental entire algebroid function defined by an irreducible equation

$$F(z, f) \equiv f^n + A_1(z) f^{n-1} + \cdots + A_n(z) = 0,$$

where $A_1, \cdots, A_n$ are entire functions and $n=3, 4, 5$. Let $\{a_j\}_{j=0}^n$ be distinct finite numbers such that arbitrary $n-1$ functions of $\{F(z, a_j)\}_{j=0}^n$ are linearly independent and

$$\sum_{j=0}^{n} \delta(a_j, f) + \sum_{\nu=1}^{n-3} \delta(a_{j_\nu}, f) > 2n-3$$

for all $n-3$ numbers $\{a_{j_\nu}\}_{\nu=1}^{n-3}$ of $\{a_j\}_{j=0}^n$. Then there exists at least one Picard's exceptional value in $\{a_j\}_{j=0}^n$. Moreover J. Noguchi [4] showed that this result is available for all $n \geqq 2$ and in the case of $n=5$, he obtained a better result.

In this note, we will discuss the case of transcendental system of entire functions and give an extension of the above fact. In the proof of Theorem 1, methods of J. Noguchi are used.

The author is grateful to Professor N. Toda for suggesting the above problems and his valuable advices.

2. Preliminaries.

Let $f_0, \cdots, f_l$ be entire functions and $X=\{F_i\}_{i=0}^N$ $(l \leqq N \leqq \infty)$ a set of linear combinations of $f_0, \cdots, f_l$ with constant coefficients. We say that X is a regular family of linear combinations of $f_0, \cdots, f_l$ when the matrices of the coefficients $(a_{i_n j})_{j=0,\cdots,l}^{n=0,\cdots,l}$ are regular for all $l+1$ integers $\{i_n\}_{n=0}^l$ $(0 \leqq i_n \leqq N)$. And we say that the elements $\{G_k\}_{k=1}^p$ in X form a basis of X if and only if $G_1, \cdots, G_p$ are linearly independent and all of X can be represented as linear combinations of $G_1, \cdots, G_p$.

Let $f=(f_0, \cdots, f_n)$ $(n \geqq 1)$ be a transcendental system in $|z|<\infty$. Namely $f_0, \cdots, f_n$ are entire functions without common zero and $\lim_{r\to\infty} T(r, f)/\log r = \infty$, where $T(r, f)$ is the characteristic function of f defined by Cartan [1], i.e. $u(re^{i\theta}) = \max_{0 \leqq j \leqq n} \log |f_j(re^{i\theta})|$ and $T(r, f)$