129. *Fundamental Solution of Partial Differential Operators of Schrödinger's Type.* I

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§ 1. **Preliminaries.** Let $ds^2=\sum_{ij}^{n} g_{ij}(x)dx_i dx_j$ be a Riemannian metric on R^n. The Laplacian $\Delta=\frac{1}{\sqrt{g}}\sum_{ij}\frac{\partial}{\partial x_i}\left(\sqrt{g}\,g^{ij}\frac{\partial}{\partial x_j}\right)$ associated with this metric naturally defines a self-adjoint operator in $L^2(R^n)$ with respect to the measure $\sqrt{g}\,dx$. This generates a one parameter group of unitary operators $U_t=\exp\frac{1}{2}i\nu^{-1}\Delta t, \nu>0, t\in R$. For any f in the domain of Δ, the function $u=U_t f$ satisfies the following equations

$$\left(i\nu\frac{\partial}{\partial t}+\frac{1}{2}\Delta\right)u=0 \qquad \text{for any } t\in R, \tag{1}$$

$$s-\lim_{t\to 0} U_t f=f. \tag{2}$$

The aim of this note is to construct, under assumptions in §§ 2 and 3, the distribution kernel $U(t,x,y)$ of the operator U_t. Our proof follows Feynman's idea [2]. Combining technique of Calderòn-Vaillancourt with method of oscillatory integrals [4], we can give rigorous mathematical reasoning to Feynman's idea.

§ 2. **Parametrix.** Let us denote by $q=q(t,y,\eta)$ and $p=p(t,y,\eta)$ the solution of the Hamiltonian equations

$$\frac{dq}{dt}=\frac{\partial H}{\partial p}, \qquad \frac{dp}{dt}=-\frac{\partial H}{\partial q} \tag{3}$$

satisfying initial conditions at $t=0$; $q=y$ and $p=\eta$, where H is the Hamiltonian function $H(q,p)=\frac{1}{2}\sum_{ij} g^{ij}(q)p_i p_j$. Since H is a homogeneous function of p's, we have

$$q(t,y,\eta)=q(1,y,t\eta) \quad \text{and} \quad tp(t,y,\eta)=p(1,y,t\eta). \tag{4}$$

Our first assumption is that

(A.I) the canonical transformation $\chi_t:(x^\circ,\eta)\mapsto(x,\xi)=(q(t,x^\circ,\eta), p(t,x^\circ,\eta))$ induces global diffeomorphism of the base space R^n.

The generating function of this canonical transformation is

$$S_0(t,x,\eta)=\int_0^t L(q,\dot{q})ds+x^\circ\cdot\eta, \tag{5}$$

where $L(q,\dot{q})$ is Lagrangean corresponding to Hamiltonian H and the