159. On the Prolongation of Local Holomorphic Solutions of Nonlinear Partial Differential Equations

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1. Introduction. Holomorphic continuation of solutions of linear partial differential equations in the complex domain has been the subject of recent investigations. (See e.g. Zerner [4] and Tsuno [3].) One of the fundamental results regarding this subject is a theorem of Zerner [4] which asserts that the solutions of a linear partial differential equation can be continued holomorphically over any non-characteristic hypersurfaces. The main purpose of this note is to present an analogous continuation theorem for general nonlinear partial differential equations. The question of the existence of noncontinuable holomorphic solutions is also studied. The complete proofs of our results will be published elsewhere.

2. The Cauchy-Kowalewsky theorem. We consider the following nonlinear system of equations for unknown functions $u_1(z), \cdots, u_N(z)$ in the complex n-space C^n:

$$(1)\qquad \frac{\partial^m u_j}{\partial z_1^m}=f_j\left(z_1, \cdots, z_n, \cdots, \left(\frac{\partial}{\partial z}\right)^\alpha u_k, \cdots\right) \qquad j=1, \cdots, N,$$

where f_j depends on the variables $z=(z_1, \cdots, z_n)$ and $(\partial/\partial z)^\alpha u_k(z)$ with multi-indices $\alpha=(\alpha_1, \cdots, \alpha_n)$, $|\alpha|\leqq m$, $\alpha_1\leqq m-1$ and $k=1, \cdots, N$. We impose the initial conditions on $u_j(z)$ on the complex hyperplane $z_1=0$ as follows.

$$(2)\qquad \begin{cases} u_j(0, z')=\phi_{j,0}(z') \\ \cdots \\ \dfrac{\partial^{m-1}u_j}{\partial z_1^{m-1}}(0, z')=\phi_{j,m-1}(z') \end{cases} \qquad j=1, \cdots, N,$$

where $z'=(z_2, \cdots, z_n)$ and $\phi_{j,k}(z')$ are arbitrarily given functions. For the regularity of f_j and $\phi_{j,k}$, we suppose that

(i) $f_j(z_1, \cdots, z_n, \cdots, p_{k,\alpha}, \cdots)$, where the variables $p_{k,\alpha}$ stand for the terms $(\partial/\partial z)^\alpha u_k$, are holomorphic on a closed polydisc $|z_\nu|\leqq r$ $(\nu=1, \cdots, n)$, $|p_{k,\alpha}|<\infty$,

(ii) $\phi_{j,k}(z')$ are holomorphic on $|z_\nu|\leqq r$ $(\nu=2, \cdots, n)$,

and set

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