89. *A Weak Solution for the Modified Frankl' Problem*

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In [2] the modified Frankl' problem for equations of mixed type was proposed and the maximum principle for the problem was proved. In this note we shall consider the system to which the Tricomi equation is reduced, construct a priori estimate by applying the ABC method [1], [3], and show the existence of a weak solution for the problem.

From the required conditions for auxiliary functions the problem must be considered on Hilbert spaces with weights which are singular on the parabolic line of the system and at a special point on it. In order to determine degrees of the weights the electronic computer FACOM 230–25 at Kumamoto University is supplementarily used. Then it can be found that such weights are restricted to peculiar ones for the system. Results for the other equations of mixed type will be published elsewhere.

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1. Modified Frankl' problem. Let Ω be a domain in the (x, y)-plane surrounded by curves $\Gamma_0, \Gamma_1, \Gamma_2, \Gamma_+$ and Γ_- as follows: Γ_0 is a segment of the x-axis located between $A(1,0)$ and $D(d,0)$, where $d>1$. Γ_2 is a curve in $y<0$ issuing from A with the slope $dx/dy=\sqrt{-y}$ (one of the characteristics of (1) below), and let the intersection of this curve and the y-axis be $C(0,-c)$. Γ_1 is a Jordan arc in $y>0$ joining D and $B(0,c)$. Γ_+ and Γ_- are segments OB and OC of the y-axis, respectively.

Let us consider the following problem for the unknown $u=(u_1, u_2)$:

$$Lu=\begin{pmatrix} y & 0 \\ 0 & -1 \end{pmatrix}\frac{\partial u}{\partial x}+\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\partial u}{\partial y}=\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \quad \text{in } \Omega, \tag{1}$$

$$\begin{cases} u_1n_2-u_2n_1=0 \text{ on } \Gamma_1,\ u_1=0 \text{ on } \Gamma_0, \\ u_2(0,y)+u_2(0,-y)=0 & \text{for } 0<y\le c, \\ u_1(0,y)+l(y)u_2(0,y)=0 & \text{for } 0<y\le c \quad \text{and} \\ u_1(0,y)+m(y)u_2(0,y)=0 & \text{for } -c\le y<0, \end{cases} \tag{2}$$

where the functions $f_1=f_1(x,y)$, $f_2=f_2(x,y)$, $l(y)$ and $m(y)$ are continuous and $n=(n_1,n_2)$ is the outer normal on Γ_1.

Let $r=\sqrt{9(x-1)^2+4|y|^3}$, and let α be a real number. Let H_α be a class of pairs of measurable functions $u=(u_1, u_2)$ with the norm