88. The Baire Category Theorem in Ranked Spaces

By Shizu NAKANISHI
University of Osaka Prefecture

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In this note, we study the Baire category theorem for a ranked space of indicator ω_0 (ω_0 is the first nonfinite ordinal). Throughout this note, the term "ranked space" will mean a ranked space of indicator ω_0. Terminologies and notations concerning ranked spaces will be the same as in [5], in particular, N will denote the set $\{0,1,2,\cdots\}$, $V(p)$, $W(p),\cdots$ preneighborhoods of p, and $V(p,n)$, $W(p,n),\cdots$ those of rank n of p.

1. The Baire category theorem. For a ranked space, we define the notion of nowhere dense as follows.

Definition 1. Let $(E,\mathcal{V})$ be a ranked space. A subset A of E is said to be *nowhere dense* in E if, for every $V(p)\in\mathcal{V}$, there exists a $V(q)\in\mathcal{V}$ such that $V(q)\subset V(p)$ and $V(q)\cap A=\phi$.

Moreover, as in [2] we define:

Definition 2. For a ranked space $(E,\mathcal{V})$, a subset A of E is said to be of *first category* if it is a countable union of nowhere dense sets. All other subsets of E are said to be of *second category*. A subset A of E is said to be *dense* in E if, for every $V(p)\in\mathcal{V}$, we have $V(p)\cap A\neq\phi$. The ranked space $(E,\mathcal{V})$ is called a *Baire space* if, for every subset A of E which is of first category, the complement $E-A$ is dense in E.

As is easily seen, if $(E,\mathcal{V})$ is a ranked space for which we can topologise E in such a way that the family of all sets belonging to $\mathcal{V}$ is a base of neighborhoods, then the notion of Baire category in $(E,\mathcal{V})$ coincides with that in the topological space E topologised in this way.

We first prove the following theorem.

Theorem 1. *Every complete ranked space is a Baire space.*

Already, for a ranked space whose indicator is an arbitrary inaccessible limit ordinal, the same theorem has been proved by K. Kunugi [2], [4] under the assumption that the family $\mathcal{V}$ of preneighborhoods in the ranked space satisfies the following conditions (B) and (C).

(B) For every $V_1(p), V_2(p)\in\mathcal{V}$, there exists a $V_3(p)\in\mathcal{V}$ such that $V_3(p)\subset V_1(p)\cap V_2(p)$.

(C) For every $V(p)\in\mathcal{V}$, if $q\in V(p)$, then there exists a $V(q)\in\mathcal{V}$ such that $V(q)\subset V(p)$.

Theorem 1 asserts that if we define nowhere dense as in Definition