84. Another Form of the Whitehead Theorem in Shape Theory

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1. Introduction. In a previous paper [5] we have established the following theorem which is a shape-theoretical analogue of the classical Whitehead theorem in homotopy theory of CW complexes.

Theorem 1.1. *Let $f:(X,x_0)\to(Y,y_0)$ be a shape morphism of pointed connected topological spaces of finite dimension. If the induced morphisms $\pi_k(f):\pi_k\{(X,x_0)\}\to\pi_k\{(Y,y_0)\}$ of homotopy pro-groups*[1] *is an isomorphism for $1\leqq k<n$ and an epimorphism for $k=n$ where $n=\max(1+\dim X,\dim Y)$, then f is a shape equivalence.*

The purpose of this note is to prove the following theorem, which corresponds to another form of the Whitehead theorem in homotopy theory of CW complexes; Theorem 1.2 was announced in a previous paper [5].

Theorem 1.2. *Let $f:(X,x_0)\to(Y,y_0)$ be the same as in Theorem 1.1. If the induced morphism $\pi_k(f):\pi_k\{(X,x_0)\}\to\pi_k\{(Y,y_0)\}$ of homotopy pro-groups is an isomorphism for $1\leqq k\leqq n$ where $n=\max(\dim X,\dim Y)$, then f is a shape equivalence.*

Furthermore, the following theorem holds.

Theorem 1.3. *Let $f:(X,x_0)\to(Y,y_0)$ be a shape morphism of pointed connected topological spaces such that the induced morphism*

$$\pi_k(f):\pi_k\{(X,x_0)\}\longrightarrow\pi_k\{(Y,y_0)\}$$

of homotopy pro-groups is an isomorphism for $1\leqq k\leqq n$. If $\dim Y\leqq n$, then there exists a unique shape morphism $g:(Y,y_0)\to(X,x_0)$ such that $fg=1$.

2. Preliminaries. Let $f:(X,x_0)\to(Y,y_0)$ be a shape morphism of pointed connected topological spaces.

As in [5], without loss of generality we may assume that $\{(X_\lambda,x_{0\lambda}),[p_{\lambda\lambda'}],\Lambda\}$ and $\{(Y_\lambda,y_{0\lambda}),[q_{\lambda\lambda'}],\Lambda\}$ are inverse systems in $\mathfrak{W}_0$ which are isomorphic to the Čech systems of (X,x_0) and (Y,y_0) respectively in pro $(\mathfrak{W}_0)$, where $\mathfrak{W}_0$ is the homotopy category of pointed connected CW complexes, and that f is an equivalence class containing a special system map

$$\{1,f_\lambda,\Lambda\}:\{(X_\lambda,x_{0\lambda}),[p_{\lambda\lambda'}],\Lambda\}\longrightarrow\{(Y_\lambda,y_{0\lambda}),[q_{\lambda\lambda'}],\Lambda\}$$

1) For the definition of the k-th homotopy pro-group of a pointed topological space (X,x_0), see [5]. Here we denote it by $\pi_k\{(X,x_0)\}$ (cf. [2]).