# 79. *Fundamental Solutions of Mixed Problems for Hyperbolic Equations with Constant Coefficients*

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§ 1. **Introduction.** We study the structure of singular supports of fundamental solutions of hyperbolic mixed problems with constant coefficients in a quarter space. Duff published a basic paper on this subject ([2]) in 1964. Although its results are precise, the paper seems to be difficult to understand. Matsumura [4] studied it by means of "Localization theorem" developed by L. Hörmander [3] and Atiyah-Bott-Gårding [1], but he did not treat the analysis of the fundamental solutions at branch points appearing in reflection coefficients. In this paper we give the "Generalized localization theorem", and by this theorem we can explain the presence of lateral waves.

We thank Prof. Matsumura for having communicated us that Wakabayashi is publishing a note on the same subject ([7]). His results are more restrictive than ours. A forthcoming paper will give detailed proofs and more precise results.

§ 2. **Notations and representation of fundamental solutions.** Let $\Omega=\{(t,x,y)\,;\,t>0,\ x>0,\ y\in R^n\}$. We consider the problem

$$(2.1)\quad\begin{cases}P(D_t,D_x,D_y)u=0 & \text{in } \Omega\\ B_j(D_t,D_x,D_y)u=0 & \text{on } \bar{\Omega}\cap\{x=0\},\ j=1,2,\cdots,\mu,\\ (u,D_tu,\cdots,D_t^{m-1}u)=(0,0,\cdots,0,i\delta_{(x-l,y)}) & \text{on } \bar{\Omega}\cap\{t=0\},\end{cases}$$

where i) $D_t=-i\partial_t$, $D_x=-i\partial_x$, $D_y=-i(\partial_{y_1},\partial_{y_2},\cdots,\partial_{y_n})$, ii) $l>0$, and iii) $P$ and $B_j$ $(j=1,2,\cdots,\mu)$ are homogeneous differential operators of degree $m$ and $m_j$ $(j=1,2,\cdots,\mu)$ with constant coefficients. We assume

( A.I ) *$P$ is strictly hyperbolic with respect to $t$,*

( A.II ) *$x=0$ is not characteristic with respect for $P$,*

(A.III) *The mixed problem* (2.1) *is $\mathcal{E}$-well posed.*

The characterization of $\mathcal{E}$-well posedness was given by Sakamoto [5]. We write the dual coordinates of $(t,x,y)$ by $(\sigma,\xi,\eta)\in R^{n+2}$, and put $\tau=\sigma-i\gamma$ $(\gamma>0)$. From (A.I), there exists no real zero of $P(\tau,\xi,\eta)$ with respect to $\xi$ for $\tau=\sigma-i\gamma$ $(\gamma>0)$, $(\sigma,\eta)\in R^{n+1}$. From (A.III), the number of roots of $P$ with positive imaginary parts is equal to $\mu$. Therefore we can represent $P$ as follows:

$$\begin{aligned}P(\tau,\xi,\eta)&=\text{const}\prod_{j=1}^{\mu}(\xi-\xi_j^+(\tau,\eta))\cdot\prod_{j=1}^{m-\mu}(\xi-\xi_j^-(\tau,\eta))\\&=\text{const}\,P_+(\tau,\eta\,;\,\xi)\cdot P_-(\tau,\eta\,;\,\xi)\end{aligned}$$

where Im. $\xi_j^{\pm}(\tau,\eta)\gtrless 0$. Here we define the matrix $L(\tau,\eta)$ by