76. *Continuity of Homomorphism of Lie Algebras of Vector Fields*

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1. Introduction. For any smooth manifold M, let $\mathcal{A}(M)$ be the (infinite dimensional) Lie algebra formed by all the smooth vector fields on M under the usual bracket operation and Diff (M) the group formed by all the diffeomorphisms of M. In [3] (Theorem 1.3.2) H. Omori proved that if M and N are compact and $\varphi: \mathcal{A}(M) \to \mathcal{A}(N)$ is a Lie algebra homomorphism which is continuous in the C^∞-topology, then φ induces a local homomorphism Diff $(M) \to$ Diff (N) as in the finite dimensional case. In this theorem the assumption of the continuity can be omitted, i.e. we can prove the following

Theorem. *Any homomorphism $\varphi: \mathcal{A}(M) \to \mathcal{A}(N)$ is continuous in the C^∞-topology.*

Since it can be shown that if φ is non-trivial and N is compact then M is also compact, we have

Corollary. *If N is compact then φ induces a local homomorphism* Diff $(M) \to$ Diff (N).

It is known that if φ is an isomorphism, then M and N are diffeomorphic, in other words, the Lie algebra $\mathcal{A}(M)$ determines the manifold M ([4], for non-compact case [2]). In case of the general homomorphism, the relation of M and N is given as follows. For any positive integer l, let M_l be a smooth manifold formed by all the sets of distinct l points of M and put $N_0 = \{q \in N \mid \varphi(X)$ vanishes at q for any $X \in \mathcal{A}(M)\}$. Then N is a finite disjoint union of subsets $N_0, N_1, \cdots, N_k$ and if N is compact then each N_l is a (topological) fibre bundle over M_l. This bundle is closely related to the jet bundle of the tangent bundle of $M^l = M \times \cdots \times M$. (It seems that $N_0 = \phi$ and N_l is a smooth bundle whose fibre is a smooth manifold with corner.) The details will appear elsewhere.

2. Sketch of the proof of Theorem. Recall that the C^∞-topology of $\mathcal{A}(M)$ is given by seminorms $|\cdot|_{U,r}$ defined as follows. Let U be a relatively compact open set of M and $(x) = (x^1, \cdots, x^n)$ a coordinate system on some neighborhood of $\bar{U}$. Then for any $X \in \mathcal{A}(M)$ with $X = \sum f^i(x)\partial_{x^i}$ on U, we put

$$|X|_{U,r} = \sup_{x \in U, |\alpha| \leqq r, i \leqq n} |D^\alpha f^i(x)|$$