136. *Periodic Linear Systems and a Class of Nonlinear Evolution Equations*

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(Comm. by Kinjirô KUNUGI, M. J. A., Oct. 13, 1975)

1. Periodic systems. Consider a linear periodic system with t regarded as a parameter:

(1.1) $$\frac{d\varphi}{dx}=A(x,t)\varphi, \qquad -\infty<x,t<+\infty,$$

where $\varphi=\varphi(x,t)$ is a complex n-column vector and $A(x,t)$ is a complex $n\times n$ matrix function. We assume that $A(x,t)$ is infinitely differentiable and periodic in x with period ω. Below, we are always in the category of infinite differentiability. From the well-known Floquet's theorem, we see that every fundamental matrix solution $X(x\,;t)$ of (1.1) has the form:

(1.2) $$X(x\,;t)=P(x,t)\exp(x\log B(t)/\omega),$$

where $P(x,t)$ is a complex nonsingular $n\times n$ matrix function which is periodic in x with period ω and $B(t)$ is a complex nonsingular $n\times n$ matrix function which does not depend on x. $B(t)$ is called a monodromy matrix of (1.1) for $X(x,t)$. The eigenvalues $\rho_j(t)$ of a monodromy matrix of (1.1) are called the characteristic multipliers of (1.1). For any fixed t, every monodromy matrix of (1.1) is similar to each other. Hence, so long as t is fixed, the characteristic multipliers and their algebraic and geometric multiplicities (which we shall call the internal structure of a monodromy matrix) do not depend on the particular fundamental solution used to define the monodromy matrix. As is well known, in order that all solutions of (1.1) are bounded in the whole axis $-\infty<x<+\infty$, it is necessary and sufficient that all characteristic multipliers of (1.1) have modulii$=1$ and have simple elementary divisors (see Hale [1]). As t varies, the internal structure of $B(t)$ may change, that is, the qualitative properties of solutions of (1.1) may change. We now propose the following question: To find $A(x,t)$ for which the equation (1.1) admits a monodromy matrix which does not depend on t, that is, the internal structure of every monodromy matrix of (1.1) does not depend on t. For this question we have

Theorem 1. *There exists a monodromy matrix of* (1.1) *which does not depend on t if and only if there exists a matrix function* $\Gamma(x,t)$ *which is defined on* $-\infty<x,t<+\infty$, *periodic in* x *with period* ω *and satisfies*