3. *Riemannian Manifolds Admitting Some Geodesic. II*[*)]

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1. **Introduction.** In this note we shall show some types of manifold which admits an isometry invariant geodesic. Let M be a Riemannian manifold and f be an isometry of M, then a geodesic α is called f-invariant geodesic if $f\alpha=\alpha$. The problem of the existence of isometry invariant geodesic is proposed by K. Grove ([1]) and in case of connected, simply connected and compact manifold a criterion of this problem is shown by him. We reform it into a calculable form ([3]) and here actually show some types of manifold which admits an invariant geodesic. The existence of an invariant geodesic is known for all compact connected, simply connected orientable manifolds of odd dimension and for its orientation preserving isometry ([3]). And so here we are concerned with even dimensional manifold.

Here the manifold M which we consider is following

[H] *compact, connected and orientable and its fundamental group is finite.*

An order of an isometry f is defined by the minimal integer n such that f^n is homotopic to the identity and denoted by ord (f). And a rank of the k-th homology group $H_k(M,Z)$ over the integer group Z is denoted by rank $H_k(M,Z)$. Then our main results are following;

Theorem A. *Let M be a $2k$-dimensional manifold of* [H] *and f be an orientation preserving isometry ($k>1$). If rank $H_k(M,Z)=2$ and* ord $(f)\not\equiv 0 \pmod 3$, *then there exists an f-invariant geodesic. If rank $H_k(M,Z)=3$,* ord $(f)\not\equiv 0 \pmod 2$ *and f has no eigenvalue* 1, *then there exists an f-invariant geodesic.*

Theorem B. *Let M be a $2k$-dimensional manifold of [H] and f be an orientation preserving isometry ($k>1$). If rank $H_k(M,Z)=$ even and* ord $(f)=2$, 4 *or* 8, *then there exists an f-invariant geodesic.*

For two dimensional manifold of [H] we can prove that there exists an f-invariant geodesic for each orientation preserving isometry f by using our result of [3].

2. **Lemmas.** Let M be a Riemannian manifold of $2k$-dimension and f be an isometry of M. Then a trace of an induced homomorphism $f_k: H_k(M,Z)\to H_k(M,Z)$ of k-th homology group which is defined by a

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