49. Some Results on Additive Number Theory. I

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In his previous papers [2]–[5], the author gave some generalizations of the theorem of Erdös and Kac in [1]. In this note we shall give some theorems on additive number theory which are obtainable by similar methods as in the above papers. The detailed proofs will be given elsewhere.

Let k be an integer >1; let l_i $(i=1,\cdots,k)$ be positive integers, and put $l_0=l_1+\cdots+l_k$.

Theorem 1. *Let P_{ij} $(i=1,\cdots,k;\ j=1,\cdots,l_i)$ be sets, each consisting of prime numbers, subject to the following conditions:*

(C_1) *For each $i=1,\cdots,k$, the sets P_{ij} $(j=1,\cdots,l_i)$ are pairwise disjoint;*

(C_2) *As $x\to\infty$,*

$$\sum_{p\leqq x,\,p\in P_{ij}}\frac{1}{p}=\lambda_{ij}\log\log x+o(\sqrt{\log\log x})$$

with positive constants λ_{ij} for $i=1,\cdots,k;\ j=1,\cdots,l_i$. (The sets P_{ij} with distinct i's need not be disjoint, and $P_{i1}\cup\cdots\cup P_{il_i}$ may not contain all primes.)

For a positive integer n, we denote by $\omega_{ij}(n)$ the number of distinct prime factors of n belonging to the set P_{ij}.

Let E be a Jordan-measurable set, bounded or unbounded, in the Euclidean space R^{l_0} of l_0 dimensions. For sufficiently large integer N, let $A(N;E)$ denote the number of representations of N as the sum of k positive integers: $N=n_1+\cdots+n_k$ such that the point $(x_{11},\cdots,x_{1l_1},\cdots,x_{k1},\cdots,x_{kl_k})$ belongs to the set E, where

$$x_{ij}=\frac{\omega_{ij}(n_i)-\lambda_{ij}\log\log N}{\sqrt{\lambda_{ij}\log\log N}} \tag{1}$$

for $i=1,\cdots,k;\ j=1,\cdots,l_i$. Then, as $N\to\infty$, we have

$$A(N;E)\sim\frac{N^{k-1}}{(k-1)!}(2\pi)^{-(l_0/2)}\int_E\exp\left(-\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{l_i}x_{ij}^2\right)dx_{11}\cdots dx_{kl_k}. \tag{2}$$

Theorem 2. *Let the polynomials $f_{ij}(\xi)$ $(i=1,\cdots,k;\ j=1,\cdots,l_i)$ of positive degree be subject to the following conditions:*

(C_1) *Each $f_{ij}(\xi)$ has rational integral coefficients, the leading coefficient being positive;*