91. The Existence and Uniqueness of the Solution of Equations Describing Compressible Viscous Fluid Flow in a Domain

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1. Introduction. The compressible viscous isotropic Newtonian fluid motion is described as follows: (the summation convention is used)

$$\frac{D\rho}{Dt}=-\rho\frac{\partial v_k}{\partial x_k}, \tag{1.1}$$

$$\begin{aligned}\frac{Dv_i}{Dt}=&\frac{1}{\rho}\frac{\partial}{\partial x_i}\left(\mu'\frac{\partial v_k}{\partial x_k}\right)+\frac{1}{\rho}\frac{\partial}{\partial x_k}\left[\mu\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)\right]\\&-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+f_i \qquad (i=1,2,3),\end{aligned} \tag{1.2}$$

$$\frac{DS}{Dt}=\frac{1}{\rho\theta}\frac{\partial}{\partial x_k}\left(\kappa\frac{\partial\theta}{\partial x_k}\right)+\frac{\mu}{2\rho\theta}\left(\frac{\partial v_j}{\partial x_k}+\frac{\partial v_k}{\partial x_j}\right)^2+\frac{\mu'}{\rho\theta}\left(\frac{\partial v_k}{\partial x_k}\right)^2, \tag{1.3}$$

(ρ, density; v, velocity; μ, coefficient of viscosity; μ', second coefficient of viscosity; κ, coefficient of heat conduction; p, pressure; f, outer force; S, entropy; θ, absolute temperature; $D/Dt=\partial/\partial t+v_k\cdot\partial/\partial x_k$).

By the physical requirements, μ,μ',κ,p and S are considered to be functions of ρ and θ such that

$$\mu'+\frac{2}{3}\mu\geqq 0;\ \mu,\kappa,p,S_\theta>0. \tag{1.4}$$

If S is smooth, then it follows from (1.1) and (1.3) that

$$\begin{aligned}\frac{D\theta}{Dt}=&\frac{1}{\rho\theta S_\theta}\frac{\partial}{\partial x_k}\left(\kappa\frac{\partial\theta}{\partial x_k}\right)+\frac{\mu}{2\rho\theta S_\theta}\left(\frac{\partial v_j}{\partial x_k}+\frac{\partial v_k}{\partial x_j}\right)^2\\&+\frac{\mu'}{\rho\theta S_\theta}\left(\frac{\partial v_k}{\partial x_k}\right)^2+\frac{\rho S_\rho}{S_\theta}\frac{\partial v_k}{\partial x_k}.\end{aligned} \tag{1.3'}$$

We shall consider a first initial-boundary value problem of (1.1), (1.2) and (1.3′) with the initial-boundary conditions:

$$\begin{cases}v(x,0)=v_0(x),\ \theta(x,0)=\theta_0(x),\ \rho(x,0)=\rho_0(x) & (x\in\Omega),\\ v(x,t)=0,\ \theta(x,t)=\theta_1(x,t) & ((x,t)\in\Gamma_T),\end{cases} \tag{1.5}$$

(Ω is a bounded or unbounded domain in R^3, whose boundary Γ belongs to $C^{2+\alpha}$ and satisfies Lyapunov conditions (cf. [4]); $\Gamma_T=\Gamma\times[0,T]$). We assume that the compatibility conditions hold and that in (1.5)

$$\begin{cases}v_0,\theta_0\in H^{2+\alpha}(\bar\Omega),\ \rho_0\in H^{1+\alpha}(\bar\Omega),\ 0<\bar\rho_0\leqq\rho_0\leqq\bar{\bar\rho}_0<+\infty,\\ 0<\bar\theta_0\leqq\theta_0\leqq\bar{\bar\theta}_0<+\infty,\ \theta_1\in H^{2+\alpha}(\Gamma_T),\ \mu,\mu',\kappa,p,S\in\mathcal{O}^{2+L}_{\mathrm{loc}}(\mathcal{D}_{\rho,\theta}),\\ f\in B^{1+L}(\bar Q_T),\end{cases} \tag{1.6}$$