113. *Degenerate Elliptic Systems of Pseudodifferential Equations*

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Introduction. Let M be a compact C^∞ manifold and $A=(a_{ij})_{i,j=1,\cdots,m}$ be a matrix of pseudodifferential operators on M whose symbols, represented by local coordinates, have homogeneous asymptotic expansions (cf. Seeley [4]). Let us consider the equation $Au=f$ on M when A is elliptic outside a C^∞ submanifold M_0 and degenerate on M_0. In the present paper we shall study the normal solvability and the subelliptic estimates for a class of equations such that $\det A_0$ (A_0 is the principal symbol of A) has multi-characteristics, while Èskin in [1] has investigated these problems in the case where $\det A_0$ is of principal type. Finally we shall give an example as an application to non-coercive boundary value problems of fourth order.

1. Assumptions and the main theorem. Let the order of a_{ij} be s_i+t_j ($s_i, t_j \in \boldsymbol{R}$), then A is a continuous operator from $\prod_{j=1}^m H_{s+t_j}(M)$ to $\prod_{i=1}^m H_{s-s_i}(M)$ ($H_s(M)$ denotes the Sobolev space on M of order s). Let M ($n=\dim M\geqq 2$) be separated into two connected components by a C^∞ submanifold M_0. We assume that the ellipticity of A is degenerate on M_0 in the following way.

Let $\{x^i=(x_0^i, \cdots, x_{n-1}^i)\}_{i=1,\cdots,N}$ be a set of local coordinates covering a neighborhood of M_0 and expressing M_0 by the equation $x_0^i=0$, and the transition from x^i to x^j in the domain where both x^i and x^j are defined be given by the form $x_0^j=x_0^i$, $x_k^j=\varphi_k^j(x_1^i, \cdots, x_{n-1}^i)$, ($k=1, \cdots, n-1$). When A is locally represented in $x^i=(t, y)=(t, y_1, \cdots, y_{n-1})$ ($i=1, \cdots, N$), its principal symbol $A_0(t, y; \tau, \eta)$ satisfies the assumptions (I) $\sim$ (IV):

(I) $\det A_0(t, y; \tau, \eta)\neq 0$ when $t\neq 0$ & $|\tau|+|\eta|\neq 0$ or $t=0$ & $\tau\neq 0$;

(II) $A_0(0, y; 0, \eta)=[0]$ (zero-matrix);

(III) $\det \partial A_0/\partial\tau(0, y; 0, \eta)\neq 0$, $|\eta|\neq 0$;

(IV) Set $\tilde{A}_0(t, y; \eta')=\partial A_0/\partial\tau(t, y; 0, \eta')^{-1}\cdot A_0(t, y; 0, \eta')(\eta'=\eta/|\eta|)$. There exist positive integers $k_1, \cdots, k_l$ such that the following decomposition of $\tilde{A}_0$ is possible: $t^{-k_1}\tilde{A}_0(t, y; \eta')$ is smooth on $t=0$ and has simple eigenvalues $\lambda_1^1(t, y; \eta'), \cdots, \lambda_{m_1}^1(t, y; \eta')$ with non-vanishing imaginary parts. Other eigenvalues all vanish as $t\to 0$. Let $P_j^1(t, y; \eta')$ be the projection $(2\pi i)^{-1}\oint(\lambda-t^{-k_1}\tilde{A}_0)^{-1}d\lambda$ for the eigenvalue $\lambda_j^1(t, y; \eta')$. Next for $t^{-k_2-k_1}\tilde{A}_0(I-\sum_{j=1}^{m_1}P_j^1)$ the same statements hold. We can