130. *Factorizations and Fundamental Solutions for Differential Operators of Elliptic-Hyperbolic Type*

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§ 0. **Introduction.** In this note we shall study an operator of the form

(0.1) $$L=D_t^m+A_1(t)D_t^{m-1}+\cdots+A_m(t) \quad \text{on } [0,T]\times R^n$$

($m\geqq 1, 0<T<\infty$), where $A_j(t)=a_j(t,X,D_x)\in\mathcal{B}_t(\$^j)$ on $[0,T]$ ($j=1,\cdots,m$) (For notations see, for example, Kumano-go [5]). We define the symbol $\sigma(L)=l(t,x,\lambda,\xi)$ for L by

(0.2) $$l=\lambda^m+a_1(t,x,\xi)\lambda^{m-1}+\cdots+a_m(t,x,\xi).$$

We call a symbol $l'=\lambda^m+b_1\lambda^{m-1}+\cdots+b_m$ ($b_j\in\mathcal{B}_t(S^j)$ on $[0,T]$) the principal symbol (or part) of L (or l), when we can write $l-l'=\sum_{j=1}^m r_j\lambda^{m-j}$ for $r_j\in\mathcal{B}_t(S^{j-1})$ on $[0,T]$, $j=1,\cdots,m$.

The starting point of the present note is the following factorization theorem, which can be proved by using Sylvester's determinant.

Theorem 0. *If the roots $\{\tau_j(t,x,\xi)\}_{j=1}^m$ of $l=0$ are separated into two groups $\{\tau_{1k}\}_{k=1}^{m_1}$ and $\{\tau_{2k}\}_{k=1}^{m_2}$ ($m=m_1+m_2$) so that $|\tau_{1k}-\tau_{2k'}|\geqq C|\xi|$ ($|\xi|\geqq M$) for any k,k' ($C>0, M>0$), then L is factorized into the form*

(0.3) $$L=L_1L_2+\sum_{j=1}^m R_j^{(-\infty)}D_t^{m-j} \quad \text{on } [0,T]\times R^n$$

(which is denoted by $L\equiv L_1L_2$ on $[0,T]$), where $R_j^{(-\infty)}\in\mathcal{B}_t(\$^{-\infty})$, and L_j ($j=1,2$) are operators of order m_j such that the principal symbols of L_j are $\prod_{k=1}^{m_j}(\lambda-\tau_{jk}(t,x,\xi))$.

In § 1 we shall discuss the Levi condition for L, and construct the fundamental solution $E(t,s)$, which is represented by Fourier integral operators, when L has the form $L\equiv L^{(+)}L^{(0)}$ (see Theorem 1.3). Then, the Cauchy problem for L can be solved in the spaces H_s, $\mathcal{B}$, etc., and the wave front set of the solution can be described through phase functions. Our results are regarded in some sense as global versions of those, obtained by Lax-Nirenberg [8] and Chazarain [1], [2], to R^n. We note also that our results can be easily (micro-) localized by considering aEb for appropriate $a,b\in\mathcal{B}_t(\$^0)$ and applying the asymptotic formula for $\sigma(aEb)$ given in [5], which states the canonical relation between a and b.

§ 1. **Main theorems.** In what follows we assume that the principal part l' of l has the form $l'=l^{(-)}l^{(+)}l^{(0)}$ where the roots $\{\tau_j^{(\pm)}\}_{j=1}^{m_\pm}$ of $l^{(\pm)}=0$