7. *Quadruply-Transitive Permutation Groups Whose Four-Point Stabilizer is a Frobenius Group*

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1. Introduction. In this paper we shall prove the following result.

Theorem. *Let G be a 4-fold transitive group on $\Omega=\{1,2,\cdots,n\}$. If the stabilizer of four points in G is a Frobenius group, then G is one of the following groups*: S_7, A_8 *or* M_{23}.

We shall use the same notations as in [1].

2. Proof of the theorem. Let K be the Frobenius kernel of G_{1234} and H a Frobenius complement of G_{1234}.

By a theorem of M. Hall, the order of G_{1234} is even.

Let P be a Sylow 2-subgroup of G_{1234}. Then $P\neq 1$. If P is isomorphic to a subgroup of H, then G is S_7 by Theorem 1 in [2]. Hence we may assume that P is contained in K. Thus P is a normal subgroup of G_{1234}.

By [1; IV] and Lemma 1 in [1; II], $|I(G_{1234})|=4$ and $|I(P)|=4,5,6,7$ or 11. If $|I(P)|\geqq 6$, then G is M_{23} by [1; VIII, IX, XI]. If $|I(P)|=5$, then $|I(G_{1234})|=5$, which is a contradiction. Hereafter we assume $|I(P)|=4$, and so, that n is an even integer.

If P is semiregular on $\Omega-I(P)$ or P is abelian, then G is A_8 by [1; VII, X]. From now on, we shall examine the case where P is neither semiregular on $\Omega-I(P)$ nor abelian, and prove eventually that this case does not arise.

Let R be a Sylow 3-subgroup of G_{1234}. By [1; XIII] and [3], R is a nonidentity group and $[R,P]\neq 1$. If R is contained in K, then $[R,P]=1$, which is a contradiction. Hence R must be contained in a conjugate of H.

Let r be an element of order three of R. Then r is an element of order three acting fixed point free on $P-\{1\}$. Hence by [4], the nilpotency class of P is two.

By Theorem A in [5], G_{123} has either (1) an abelian normal subgroup $\neq 1$, or (2) a unique minimal normal subgroup, and this minimal normal subgroup is simple. In the case (1), G must be S_7 or M_{23} by [6], in contradiction to our present assumption $|I(P)|=4$. We shall now consider the case (2). Let N be the minimal normal subgroup of G_{123}. It is easily seen that G_{123} is contained in $Aut(N)$.