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1. *Approximation of an Irrational Number by Rational Numbers.*

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Let ω be any positive irrational number, whose expansion into simple continued fraction is represented by

$$\omega = a_0 + \frac{1}{a_1 +} \frac{1}{a_2 +} \cdots\cdots \frac{1}{+ a_n +} \cdots\cdots = [a_0 a_1 a_2 \cdots\cdots a_n \cdots].$$

If $\frac{P_n}{Q_n} = [a_0 a_1 a_2 \cdots\cdots a_n]$ be the n-th convergent, and

$$S_n = Q_n^2 \left| \omega - \frac{P_n}{Q_n} \right|,$$

then the classical theorem due to HURWITZ and BOREL can be expressed by

$$\text{Mini}\,(S_{n-1}, S_n, S_{n+1}) < \frac{1}{\sqrt{5}},$$

and

$$\text{Mini}\,(S_{n-1}, S_n, S_{n+1}) < \frac{1}{\sqrt{8}}, \text{ if } a_{n+1} = 2.$$

I have extended[1] this theorem in the form

$$\text{Mini}\,(S_{n-1}, S_n, \cdots\cdots, S_{n+3}) < \frac{5}{\sqrt{221}},$$

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1) Bemerkung zur Theorie der Approximation der irrationalen Zahl durch rationale Zahlen, Science Reports of the Tôhoku Imperial University, Ser. I, **14** (1924). See also the Japanese Journal of Mathematics, **1** (1924).