103. *On the Projective Differential Geometry of Plane Curves and One-Parameter Families of Conics.*

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1. Recently the projective differential geometry has been developed by many mathematicians, but they all considered points, lines or planes as elements. From this fact the projective differential invariants have lost, as we can readily see, their geometrical meaning not a little. In this paper I will develope the theory of projective differential geometry of plane curves and one-parameter families of conics by considering conics as elements.

2. Consider a one-parameter family of conics in the plane, and let its equation in homogeneous coordinates be

$$\sum a_{ik}(t)x_i x_k = 0.$$

We assume the determinant $|a_{ik}(t)|$ not identically zero, and normalize $a_{ik}(t)$ as follows:

$$\overset{*}{a}_{ik}(t) = |a_{ik}(t)|^{-\frac{1}{3}} a_{ik}(t)$$

which we consider as coordinates of the family of conics. In the following we assume a_{ik} as normalized. Apply a projective transformation

$$\overline{x}_i = a_{ij} x_j$$

to the family, then we have

$$|\overline{a}_{ik}(t)| = |a_{ik}(t)|\,|a_{ij}|^2.$$

so that we can assume without any loss of generality that

$$|a_{ij}| = \pm 1.$$

3. Next we introduce the following notation:

$$(l, m, n) = \frac{1}{2}\sum \begin{vmatrix} a_{11}^{(l)} & a_{12}^{(m)} & a_{13}^{(n)} \\ a_{21}^{(l)} & a_{22}^{(m)} & a_{23}^{(n)} \\ a_{31}^{(l)} & a_{32}^{(m)} & a_{33}^{(n)} \end{vmatrix}$$