7. *Asymmetric Vibrations.*

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(Rec. Nov. 20, 1926. Comm. Jan. 12, 1927.)

Asymmetric vibrations, which are often found in motions of imperfectly elastic bodies, was first treated by Helmholtz[1] in his theory of combination tones, and by many physicists with special relation to acoustics,[2] optics and thermal expansion.[3] The solution is mostly approximative, as the imperfection of elastic property is supposed to be very small, but a more complete solution is sometimes desirable, when the deviation from Hooke's law is considerable.

The general aspect of the problem can be presented in the following form. Suppose a particle to be acted upon by an attractive force F(r) depending on distance r. If it be displaced by ξ

$$F(r+\xi) = F(r)+\xi\frac{\partial F}{\partial r}+\frac{\xi^2}{2}\frac{\partial^2 F}{\partial r^2}+\cdots\cdots \tag{1}$$

If the restitutive force be simply proportional to the displacement, the vibration is simple harmonic, but if $\xi^2\frac{\partial^2 F}{\partial r^2}$ be not negligible, the vibration becomes asymmetric. In imperfectly elastic bodies, we have to retain ξ^2 and higher powers, especially when the amplitude is not small; a most remarkable example is presented by rock materials constituting the earth's crust. In addition to this, they show plasticity, which is of great importance in many problems of geophysics.

To simplify the calculation, consider a particle displaced by ξ from its position of equilibrium; then the equation of motion is

$$m\frac{d\xi}{dt^2} = -f\xi+g\xi^2, \tag{2}$$

retaining only the first two terms of the restitutive force; f and g are constants measured by the differential coefficients of (1). The first integral leads to

$$\frac{m}{2}\left(\frac{d\xi}{dt}\right)^2 = a-f\frac{\xi^2}{2}+g\frac{\xi^3}{3} \tag{3}$$