90. *On the Order of the Absolute Values of a Linear Form, (Third Report.)*

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1. In two Notes published in these Proceedings[1] I have reported on my researches on the order of $|\alpha x-y+\beta|$. Especially, in the second of these notes I have given the upper limit of $\liminf |x(\alpha x-y+\beta)|$ for some classes of the irrational number α. Since then I have got for some cases the more precise and best possible limit, which will be shown in the following lines.

In the case iii *l.c.*, that is, when $\liminf q_i=2k$, we have the following theorem :

If $k\geqq 3$ and $\alpha x-y-\beta$ is not equivalent to

$$\sqrt{k^2+1}\,x-y+\frac{\sqrt{k^2+1}-k+1}{2}\,,$$

then

$$\liminf |x(\alpha x-y+\beta)|\leqq\frac{1}{4\left\{\dfrac{1}{1+\omega}+\dfrac{1}{2k-1}+\dfrac{1}{2k+\omega}\right\}}\,, \qquad \text{(A)}$$

where
$$\omega=\frac{1}{2k+2}+\frac{1}{2k+4}+\frac{1}{2k+2}+\frac{1}{2k+4}+\cdots\cdots, \qquad \text{if } k\geqq 6$$

$$=\frac{1}{2k+2}+\frac{1}{2k+6}+\frac{1}{2k+2}+\frac{1}{2k+6}+\cdots\cdots, \qquad \text{if } k=4, 5$$

$$=\frac{1}{8}+\frac{1}{16}+\frac{1}{8}+\frac{1}{16}+\cdots\cdots, \qquad \text{if } k=3.$$

The equality in (A) occurs for infinitely many (non-enumerable) forms, and if ε is any positive number smaller than the right-hand side

1) On the extension of Klein's geometrical interpretation of continued fraction, these Proc. 2, 100. On the extension of a theorem of Minkowski, ibid. 2, 305.