111. On Transcendental Numbers.

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The following theorem was proved by Kempner.[1]

Let a be an integer greater than 1 ; α_n $(n=0, 1, 2,\dots\dots)$ any positive or negative integer smaller in absolute value than a fixed arbitrary number M, but only a finite number of the α_n equal to 0, then

$$f(x)=\sum_{n=0}^{\infty}\frac{\alpha_n}{a_n}x^n, \qquad a_n=a^{2^n},$$

represents a transcendental number for any rational number x.

As Blumberg[2] has shown, the condition that only a finite number of coefficients α_n shall be zero may be removed, so that

$$f_1\left(\frac{p}{q}\right)=\sum\frac{\alpha_{\sigma_n}}{a'_n}\left(\frac{p}{q}\right)^n, \qquad a'_n=a^{2^{\sigma n}}$$

represents a transcendental number, when $\sigma_1<\sigma_2<\dots\dots<\sigma_n\rightarrow\infty$.

He proved this theorem by distinguishing between two cases, where

(1) for every n there are two consecutive σ_n's greater than n and differing by more than k,

(2) after a certain point, the difference between two consecutive σ_n's is less than or equal to k.

In the following lines I will give a generalzation of Kempner-Blumberg's theorem, which can be proved without distinction of the two cases.

Our theorem runs as follows :

The integral transcendental function

$$f(x)=\sum_{n=0}^{\infty}\frac{\alpha_n}{a^{\sigma_n}}x^n,$$

1) Trans. American Math. Soc., **17** (1916).
2) Bulletin American Math. Soc., **32** (1926).