PAPERS COMMUNICATED

57. *On a Characteristic Property of the Sections of Some Power Series.*

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Theorem. Let

(1) $$f_{km}(x)=1+a_1x+a_2x^2+\ldots\ldots+a_{km}x^{km}$$

be the km-th section of a power series

(2) $$f(x)=1+a_1x+a_2x^2+\ldots\ldots+a_nx^n+\ldots\ldots$$

where k is a given positive integer.

If $f_{km}(x)=0$ has all their roots on $|x|=1$ for $m=1, 2,\ldots,$ then we have

(3) $$f(x)=\frac{1+a_1xe^{i\theta}+\ldots\ldots+a_{k-1}(xe^{i\theta})^{k-1}}{1-(xe^{i\theta})^k}$$

with the conditions that

(4) $$a_{k-i}=\overline{a_i}\quad\begin{pmatrix} i=1, 2,\ldots \dfrac{k}{2} & \text{when } k \text{ is even} \\ i=1, 2,\ldots \dfrac{k-1}{2} & \text{when } k \text{ is odd}\end{pmatrix}$$

and that

(5) $1+a_1x+\ldots\ldots+a_{k-1}x^{k-1}=0$ has all its roots in $|x|\geqq 1$.

And conversely.

Proof. By the hypothesis we have $|a_{km}|=1$, and we can put $a_k=1$. Further

(6) $$a_0\overline{a_\nu}=\overline{a_{km}}\,a_{km-\nu}\quad(\nu=1, 2,\ldots\ldots km)$$

Hence

$$a_{km}=1\quad m=2, 3,\ldots\ldots)\quad\text{and}\quad a_{k-i}=\overline{a_i},\ (i=1, 2,\ldots\ldots k-1).$$