177. *A Generalization of Almost Periodic Functions.*

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Introduction. The Bohr's theory[1] of almost periodic functions has its origin in the problem: What function $f(x)$ can be decomposed, in the interval $-\infty < x < \infty$, into pure oscillations, that is, into oscillations of the form $e^{i\lambda x}$? The simplest functions of this kind are the sum of a finite number of oscillations:

$$s(x) = \sum_{\nu=1}^{N} a_\nu e^{i\lambda_\nu x}.$$

Prof. Bohr adjoined the limit functions to the class (F) of such functions; we understand by a limit function $f(x)$ of the class (F), if there exists a sequence $s_1(x)$, $s_2(x)$, $s_3(x)$, ...of functions in (F), such that

$$(1) \qquad f(x) = \lim_{n=\infty} s_n(x)$$

uniformly for every x, that is

$$(1) \qquad \underset{-\infty<x<\infty}{\text{Upper Boundary}} \ |f(x)-s_n(x)| \rightarrow 0, \quad \text{as} \quad n \rightarrow \infty.$$

Any function belonging to the adjoined class (C) is called almost periodic. In (1) we can take $s_n(x)$ as an almost periodic functions, without affecting the class (C).

The theory of almost periodic functions was extended by many authors in replacing the limiting equation (1) by more general ones. It seems to me, however, that a natural extension of (1) is the mean convergence. By

$$(2) \qquad \lim_{n=\infty} s_n(x) = f(x) \qquad (C, k),$$

where $k \geqq 0$, we mean that

$$(3) \quad \text{Upper Boundary} \int_{a-T}^{a+T} \left(1-\frac{|x|}{T}\right)^k d\,|f(x)-s_n(x)| \rightarrow 0, \text{ as } n \rightarrow \infty$$

1) Bohr, zur Theorie der fastperiodischen Funktionen, I-III, Acta Math. **45-47** (1925-26).