PAPERS COMMUNICATED

1. *On a System of Generalized Orthogonal Functions.*

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Let $J_0(x)$ be the Bessel's function of order zero, and consider the system of functions consisting of

$$\varphi(x, \lambda) = \sqrt{\pi\lambda x}\ J_0(\lambda x), \quad (\lambda \geqq 0, \ x \geqq 0).$$

This forms a system of generalized orthogonal functions in the sense that I have defined in the previous paper in these Proceedings, 2 (1926), that is,

$$M\left\{\varphi(x, \lambda)\,\varphi(x, \mu)\right\} = \delta_{\lambda\mu}, \quad (\delta_{\lambda\mu} = 1 \text{ for } \lambda = \mu, \ =0 \text{ for } \lambda \neq \mu)$$

where $M\left\{f(x)\right\}$ denotes $\lim \frac{1}{T}\int_0^T f(x)\,dx$ for $T \to \infty$.

This fact follows immediately from the asymptotic formula

$$\sqrt{\pi x}\ J_0(x) = \sqrt{2}\ \cos\left(\frac{\pi}{4} - x\right) + O\left(\frac{1}{x}\right)$$

and the following

Lemma. If $f(x)$, $g(x)$ are integrable in every finite interval in $(0, \infty)$, and $M\left\{f(x)\right\}$ exists, then $M\left\{g(x)\right\} = M\left\{f(x)\right\}$, provided that $g(x) = f(x) + O\left(\frac{1}{x}\right)$.

As a generalization of the almost periodic functions of Bohr, I have treated in the previous paper a class of functions (F), uniformly approximable by a linear combination of functions in the system $\left\{\varphi(x, \lambda)\right\}$ for $x \geqq 0$, and have deduced Parseval's formula for the class (F).

Recently Prof. Wiener has deduced in his important paper, the spectrum of an arbitrary function, Proc. London Math. Society, 27 (1928), the Parseval's formula for the almost periodic functions very ingeniously by applying his theory of the spectrum of arbitrary functions. I wish here to remark that Wiener's theory can also be applicable to the class (F).