PAPERS COMMUNICATED

98. *An Extension of the Lebesgue Measure.*

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The domain of the definition of a completely additive set function $\mu(E)$ must be a closed family (σ-Körper) of sets, which I denote by $\mathfrak{F}(\mu)$. The Lebesgue measure $m(E)$ has for its domain of the definition, $\mathfrak{F}(m)$, the closed family of sets, which are measurable in Lebesgue sense. Now there is a problem: Is there any completely additive set function $\mu(E)$, whose domain of the definition $\mathfrak{F}(\mu)$ contains $\mathfrak{F}(m)$, and the value of $\mu(E)$ at any set belonging to $\mathfrak{F}(m)$, is equal to its Lebesgue measure? In this paper, I will construct such a set function $\mu(E)$.

By the Carathéodory's theory of measure,[1] $\mu^※(E)$ being a measure function, if a set A satisfies the following relation

$$\mu^※(W)=\mu^※(AW)+\mu^※(W-AW) \tag{1}$$

for any set W of finite $\mu^※$-measure, then A is said to be $\mu^※$-measurable, and the aggregate of all such $\mu^※$-measurable set being a closed family $\mathfrak{F}(\mu)$, the set function $\mu(E)$ defined in $\mathfrak{F}(\mu)$ such that

$$\mu(A)=\mu^※(A),$$

is completely additive in $\mathfrak{F}(\mu)$.

Now let $m^※(E)$ be the exterior Lebesgue measure, and consider the set function

$$\mu^※(E)=m^※(E\Omega), \tag{2}$$

where Ω is the non-measurable set, constructed in the Carathéodory's treatise,[2] which has the whole space as its same-measure cover, that is, if M be any $m^※$-measurable set of finite $m^※$-measure, then

$$m(M)=m^※(M\Omega). \tag{3}$$

Then $\mu^※(E)$ is also a measure function,[3] and we have a completely

1) Carathéodory, *Vorlesungen über reelle Functionen*, zweite Aufl. (1927), 246.
2) *Ibid.*, 354.
3) *Ibid.*, 240.