98. Connections in the Manifold Admitting Generalized Transformations.

By Tōyomon HOSOKAWA.
Mathematical Institute, Hokkaido Imperial University, Sapporo.
(Comm. by M. FUJIWARA, M.I.A., Oct. 12, 1932.)

In the present paper a general manifold is defined in which to every point of a manifold X_n is associated a system of the quantities, $\overset{(1)}{P}{}^{\nu}_{a}, \overset{(2)}{P}{}^{\nu}_{a}, \ldots\ldots \overset{(h)}{P}{}^{\nu}_{a}$. We shall develop the notions of the point transformation for this general manifold, and then by an analogous method as in a previous paper[1] the connections will be established in it.

1. *The local geometry.* Consider an n dimensional space X_n of coordinates x^{ν} ($\nu=a_1, \ldots\ldots a_n$), and to each point in X_n corresponds a system of h mutual independent quantities $\overset{(1)}{P}{}^{\nu}_{a}, \ldots\ldots \overset{(h)}{P}{}^{\nu}_{a}$, whose directions are indeterminate, and $a=1, 2, \ldots\ldots K$. We consider $\overset{(1)}{P}{}^{\nu}_{a}$ as the elements of *K-spread*,[2] depending analytically on a system of parameters (u^a ; $a=1, 2, \ldots\ldots K$). This new manifold is called *the general manifold.*

We shall now assume for the quantities $\overset{(1)}{P}{}^{\nu}_{a}, \ldots\ldots \overset{(h)}{P}{}^{\nu}_{a}$:

$$d\overset{(i)}{P}{}^{\nu}_{a}=\overset{(i)}{\Psi}{}^{\nu}_{a/\lambda}dx^{\lambda} \qquad \begin{pmatrix} i=1, 2, \ldots\ldots h \\ a=1, 2, \ldots\ldots K \end{pmatrix}, \tag{1.1}$$

where $\overset{(i)}{\Psi}{}^{\nu}_{a/\lambda}$ are arbitrary functions.

Let us consider the transformations

$$'x^{\nu}='x^{\nu}\ (x^{\nu}, \overset{(1)}{P}{}^{\nu}_{a}, \ldots\ldots \overset{(h)}{P}{}^{\nu}_{a}), \qquad \nu=a_1, \ldots\ldots a_n, \tag{1.2}$$

in the general manifold. By differentiation of (1.2), we get

$$d'x^{\nu}=\left(\frac{\partial' x^{\nu}}{\partial x^{\lambda}}+\frac{\partial' x^{\nu}}{\partial \overset{(i)}{P}{}^{\mu}_{a}}\overset{(i)}{\Psi}{}^{\mu}_{a/\lambda}\right)dx^{\lambda}. \tag{1.3}$$

We make use the usual convention for indices about every one of the letters λ, i and a.

Any set of n quantities $V^{\nu}(x^{\nu}, \overset{(1)}{P}{}^{\nu}_{a}, \ldots\ldots \overset{(1)}{P}{}^{\nu}_{a})$, ($\nu=a_1, \ldots\ldots a_n$), transformed by the transformations (1.2) into new n quantities $'V^{\nu}\ ('x^{\nu}, '\overset{(1)}{P}{}^{\nu}_{a}, \ldots\ldots '\overset{(h)}{P}{}^{\nu}_{a})$ in such a way that

1) T. Hosokawa: Science Reports, Tohoku Imp. University, **19** (1930), p. 37–51.
2) J. Douglas: Math. Annalen, **105** (1931), p. 707.