3. A Theorem on Cesàro Summability.

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(Comm. by M. FUJIWARA, M.I.A., Jan. 12, 1934.)

Recently Prof. Y. Okada proved the theorem:

Let the series[1] $\sum_{n=0}^{\infty} a_n$ *be summable* (C, a) $(a > -1)$, *with sum* s. *If, for any given non-negative* k,

$$\underline{\lim}\,(C_n^{(k)} - C_m^{(k)}) \geqq 0 \qquad \left(\frac{n}{m} \to 1, \quad n > m \to +\infty\right) \tag{1}$$

holds, then the series is summable (C, k) *with sum* s, *where*

$$C_n^{(k)} = \frac{S_n^{(k)}}{\binom{n+k}{k}},$$

and

$$S_n^{(k)} = \sum_{\nu=0}^{n} \binom{n-\nu-k}{k} a_\nu .$$

In the present paper, it is aimed to deduce a more general theorem, from Schmidt's theorem which runs as follows:

Let the series be summable by Abel's method with sum s. *If*

$$\underline{\lim}\,(s_n - s_m) \geqq 0 \qquad \left(\frac{n}{m} \to 1, \quad n > m \to +\infty\right)$$

holds, then the series is convergent with sum s, *where*

$$s_n = \sum_{\nu=0}^{n} a_\nu .$$

Theorem. Let the series $\sum a_n$ *be summable by Abel's method with sum* s. *If for any given non-negative* k, (1) *holds, then the series is summable* (C, k) *with sum* s.

Without loss of generality, we can suppose that $s=0$. Consider the series

$$\sum_{\nu=0}^{n} (C_\nu^{(k)} - C_{\nu-1}^{(k)}), \qquad (C_{-1}^{(k)} = 0),$$

then

$$\sum_{\nu=0}^{n} (C_\nu^{(k)} - C_{\nu-1}^{(k)}) = C_n^{(k)} .$$

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1) We consider here only real series.

2) Y. Okada: On the converse of the consistency of Cesaro's summability, Tohoku Mathematical Journal, **38** (1933).