PAPERS COMMUNICATED

37. *The Foundation of the Theory of Displacements, III.*

(*Application to a manifold of matrices.*)

By Akitsugu KAWAGUCHI.

Mathematical Institute, Hokkaido Imperial University, Sapporo.

(Comm. by M. FUJIWARA, M.I.A., Feb. 12, 1934.)

Three kinds of displacements for a manifold of matrices are considered in this paper from the standpoint of the general theory set out in my previous paper (F.D.I.) and also from that of its application to a manifold with a linear connection.

1. Let us consider a manifold of finite dimensions M with a coordinate system x^λ $(\lambda=1, 2,\ldots\ldots, n)$ and associate a manifold of matrices to its each point, where under the underlying isomorphism between any two manifolds $\overline{M}$ in § 6 (F.D.I.) the corresponding matrices have corresponding elements of the same values. Then (10) in F.D.I. becomes

$$\nabla A=dA+\Gamma(A) \tag{1}$$

for a matrix A in $\overline{M}$ determined uniquely for every point of M, where $\Gamma(A)$ is a matrix depending on x^λ and the differential dA is a matrix, whose elements are differentials of that of A. Our object is not to consider such a general displacement, but a special one such that

$$\nabla A=dA+\Gamma A+A\Gamma', \tag{2}$$ [1]

where Γ and Γ' are matrices independent of A. This displacement is clearly linear and has many interesting properties, as we see in the following. When Γ and Γ' are linear forms with respect to dx^λ and have such forms that $\Gamma=\Gamma_\lambda(x)dx^\lambda$, $\Gamma'=\Gamma'_\lambda(x)dx^\lambda$, then it follows from (2)

$$\nabla_\lambda A=\frac{\partial A}{\partial x^\lambda}+\Gamma_\lambda A+A\Gamma'_\lambda . \tag{3}$$

2. For the covariant derivatives of the inverse matrix A^{-1} of A it seems to be most natural to define them in the same manner as

1) We may also define such a differentiation, that $\nabla a_{ij}=da_{ij}+\sum_{k,l}\Gamma^{kl}_{ij}a_{kl}$ where $A=((a_{ij}))$. A special case of this connection has been studied by S. Hokari: Über die Bivektorübertragung, Journal of the Faculty of Science, Hokkaido Imperial University, Series I, **2** (1934), 103-117.