122. *Extension of Duhamel's Theorem.*

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Introduction. Regarding the conduction of heat we have the so-called Duhamel Theorem, which will at once give the solution when a solid initially at zero temperature, is exposed thereafter to any given *variable* temperature at the *surface*, if we know the solution for the solid whose surface is kept at a *constant* temperature other than the initial value.

Since Duhamel[1] first presented the theorem, the method of proof has been much improved by Riemann,[2] Carlslaw[3] and others, but they restricted themselves to heat-conduction corresponding to the differential equation $\frac{\partial\theta}{\partial t}=\nu\nabla^2\theta$ and the *boundary* temperature $\bar{\theta}=F(t)$.

In other branches of mathematical physics, however, no such general rule has been pronounced, and even special examples treated in similar manner are very rarely found, so far as the present writer knows.

The writer's new theorem here to be introduced, is a wide extension of Duhamel's, applicable not only to heat-conduction but also to several domains of physics and even pure mathematics. Moreover it may be used for the varying action or condition of *interior* bodily nature, as well as of boundary nature such as the surface temperature in Duhamel's theorem. The writer determines the limits within which the method similar to Duhamel theorem can be applied.

Definition. Let $x_1, x_2, \cdots\cdots, x_n$ be independent variables. Let a quantity $W(x_1, x_2, \cdots\cdots, x_n)$ be such a function as will be influenced by any other quantity $F(x_1, x_2, \cdots\cdots, x_n)$ which may be some "action" or "circumstance" in physical meaning.

Let $W_1, W_2, W_3, \cdots\cdots, W_m$ be the values of W corresponding to F's several values $F_1, F_2, F_3, \cdots\cdots, F_m$, provided that all other circumstances remain the same.

If, for the value of F

$$F=F_1+F_2+F_3+\cdots\cdots+F_m\,,$$

we have the corresponding value of W such that

$$W=W_1+W_2+W_3+\cdots\cdots+W_m\,,$$

then we define the quantity W to be *additive* or *superposable* with respect to F.

Theorem. Let a function $W(x_1, x_2, \cdots\cdots, x_n)$ be holomorphic for x_1 and additive with respect to $F(x_1, x_2, \cdots\cdots, x_n)$, and let

$$W=0\,,^{1)}\qquad F=0\qquad \text{for}\qquad x_1<0\,.$$

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1) Journ. d. L'Ecole Polytech, **14**, 20–29 (1833).
2) Partielle Diff. Gleichungen II, 102–105 (1901).
3) The Conduction of Heat, 17 (1921).