PAPERS COMMUNICATED

48. *Some Remarks on the Uniqueness of Solution of a Differential Equation.*

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I. Taking a function $f(x, y)$, continuous and limited ($\leqq M$) in the domain R

$$0 \leqq x \leqq l, \qquad -\infty < y < +\infty, \tag{1}$$

we consider the differential equation

$$y' = f(x, y) \tag{2}$$

and its solution passing through the origin $(0, 0)$, the existence of which being well known.

Many criterions were found for the uniqueness of such solution. Almost all of those criterions are established by means of an auxiliary differential equation, whose non-negative solution satisfying the initial condition

$$y(+0) = 0 \qquad y'(+0) = 0 \tag{3}$$

is known to be $y \equiv 0$ uniquely.

In the place of such auxiliary differential equation, we may take a system S of curves, which is not necessarily a system of integral curves of a simple differential equation. We only require that the system S satisfies the following conditions.

(I.) Every curve of S lies within the domain R'

$$0 < x < l, \qquad 0 < y \tag{4}$$

Every curve is simple and has continuous tangent. Its end points either approach indefinitely to the boundary $x = 0$ or l of R' or tend to infinity.

(II.) Every curve of S is so oriented that the boundary $y = 0$ of R' lies on the right hand side of the curve. Passing through any point (x, y) of R', there goes at least a curve of S. The amplitude of the oriented tangent of the curve at (x, y) is determinate. We denote it by $\theta(x, y)$.

(III.) There is no curve of S, which approaches indefinitely to the origin, touching altimately the axis of x.

Under such circumstances, we consider a non-negative differentiable function

$$y = \varphi(x), \qquad 0 \leqq x \leqq l \tag{5}$$

If $\varphi(x)$ is not identically zero, and if the curve (5) always cross the curves of S from left to right, as x increases, then the portion of the