PAPERS COMMUNICATED

1. *An Invariant Property of Siegel's Modular Function.*

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C. L. Siegel[1] recently defined the following remarkable function

$$f_r(X)=\sum_{(P,\,Q)}|PX+Q|^{-2r},^{2)}$$

where X is a quadratic matrix of the dimension n with a positive "imaginary part" and P and Q are matrices of the same dimension having rational integral components, while $\sum$ sums over all non-associated symmetrical pairs of matrices P and Q without a left common divisor.

It is absolutely and uniformly convergent when an integer $r>\frac{n(n+1)}{2}$ and represents a modular function of the nth. degree and of the dimension $-2r$.

In making use of the system of representatives of the classes of transformations of Siegel's modular group, that I have given in my former paper,[3] I will extend in this work a property of Eisentein's series, due to Mr. Hecke,[4] to this new function: namely I prove the following

Theorem: Let $T_i=\begin{pmatrix}A_i & B_i\\ 0 & D_i\end{pmatrix}$, $i=1,2,\ldots\ldots,k$ be the complete system of representatives of the classes of transformations of the degree m, then by the linear operator $\sum_{i=1}^{k}T_i|D_i|^{-2r}$ the function $f_r(X)$ is multiplied by a constant factor N;

$$\sum_i|D_i|^{-2r}f_r\big(T_i(X)\big)=Nf_r(X)\,.$$

Firstly I prove the

Lemma: The number of the classes of transformations of the degree m, $T=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$, in which A and B are two given matrices, depends only on the common divisor G which makes two matrices $G^{-1}A$ and $G^{-1}B$ left-relatively-prime.

Proof: As T is a transformation of the degree m, namely

1) C. L. Siegel, Analytische Theorie der quadratischen Formen, 1.
2) $|PX+Q|$ represents the determinant of the matrix $PX+Q$.
3) M. Sugawara. On the transformation theory of Siegel's modular group.
4) E. Hecke. Die Prinzahlen in der Theorie der elliptischen Modulfunktionen.