31. *The Method of Successive Approximation in the Old Japanese Mathematics.*

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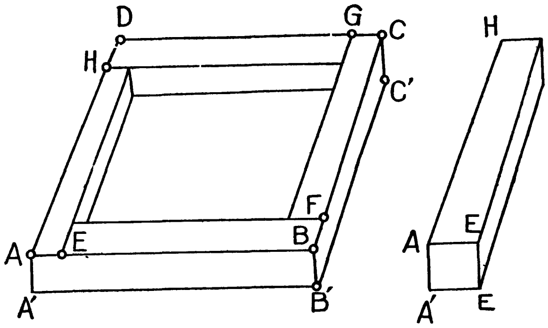
It is well known that we can find the method of successive approximations in a manuscript Kaihô-Yeizikuzitu (開方盈朒術) by *Genzyun Nakane* (中根彦循, 1701–1761), written in 1729 (享保十四年). It was remarked however by Mr. Yoshio Mikami[1] that the said method was already used by *Takakazu Seki* (關孝和) in his manuscript Daizitu-Bengi (題術辨議), the date unknown.

I wish here to report that a work Sangaku-Yenteiki (算學淵底記) or Sanpô Hutudankai (算法勿憚改) by *Murase* (村瀨義益), written in 1673 (寛文十三年), contains two problems solved by the method of successive approximations.

The first problem treats of a fireplace-frame (爐緣), which consists of 4 pieces of rectangular parallelopiped, whose breadth and height are equal. The said problem runs as follows.



Given the volume of a fireplace-frame $v=192$ and the length $AB=14$, it is required to find the length $AA'=BB'=AE=BF=\cdots\cdots$

If we denote $AA'=x$, then we have a cubic equation

$$x^2(14-x)=\frac{1}{4}\times 192=48.$$

The author of the said work gave two methods. The first starts from the form $x=\sqrt{\frac{1}{14}(48+x^3)}$ and the second from $x=\sqrt{\frac{48}{14-x}}$. Then determining $x_1, x_2, x_3, \cdots\cdots$ successively by

$$x_1=\sqrt{\frac{1}{14}(48+x_0^3)},\quad x_2=\sqrt{\frac{1}{14}(48+x_1^3)},\quad x_3=\sqrt{\frac{1}{14}(48+x_2^3)},\cdots\cdots;$$

$$x_1=\sqrt{\frac{48}{14-x_0}},\quad x_2=\sqrt{\frac{48}{14-x_1}},\quad x_3=\sqrt{\frac{48}{14-x_2}},\cdots\cdots$$

respectively (starting with $x_0=0$), we obtain

$$x_1=1.85,\quad x_2=1.97,\quad x_3=1.9936;$$
$$x_1=1.85,\quad x_2=1.9876,\quad x_3=1.99907$$

1) Tôyô-Gakuhô, vol. 21, 1934.