81. *On the General Zetafuchsian Functions.*[1]

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1. Introduction.

In a previous paper[2] I have constructed a theory of automorphic functions of higher dimensions. Using the same notations as in that paper, we will call the space $\mathfrak{A}$ the set of symmetrical matrices Z of the dimension n with the condition $E > Z'\bar{Z}$. We put now $R(Z) = -\log|E-\bar{Z}Z|$, that is $|E-\bar{Z}Z| = e^{-R(Z)}$. Then $R(Z)\to o$ or ∞ according as $Z\to O$ or Z tends to the boundary of the space $\mathfrak{A}$ and conversely.

Matrices U of dimension $2n$ satisfying the conditions $U'JU=J$, $U'S\bar{U}=S$, where $J=\begin{pmatrix} O & E \\ -E & O\end{pmatrix}$ and $S=\begin{pmatrix} E & O \\ O & -E\end{pmatrix}$ are of the form $U=\begin{pmatrix} U_1 & U_2 \\ U_3 & U_4\end{pmatrix}=\begin{pmatrix} \bar{U}_4 & \bar{U}_3 \\ U_3 & U_4\end{pmatrix}$ and form a group Γ.

When Z is an inner or a boundary point of $\mathfrak{A}$, $W=(U_1Z+U_2)(U_3Z+U_4)^{-1}$ is also an inner or a boundary point of $\mathfrak{A}$ respectively, so that we called this transformation a displacement of the space $\mathfrak{A}$ induced by $U\in\Gamma$. We then have

Lemma 1. If $W=(U_1Z+U_2)(U_3Z+U_4)^{-1}$, $U=\begin{pmatrix} U_1 & U_2 \\ U_3 & U_4\end{pmatrix}\in\Gamma$, then $\| U_3Z+U_4\|^{-2}=\dfrac{|E-\bar{W}W|}{|E-\bar{Z}Z|}$, that is $\| U_3Z+U_4\|^{-2}=e^{R(Z)-R(W)}$.

Proof. Every point of $\mathfrak{A}$ can be represented in the form $Z=PQ^{-1}$, where P, Q make a pair of symmetrical matrices with the condition $|Q|\neq 0$. Let $\begin{pmatrix} P_1 \\ Q_1\end{pmatrix}=U\begin{pmatrix} P \\ Q\end{pmatrix}$ then P_1, Q_1 make also a pair of symmetrical matrices with the condition $|Q_1|\neq 0$, such that $W=P_1Q_1^{-1}$, and

$$\begin{aligned} E-\bar{W}W &= E-\bar{Q}_1'^{-1}\bar{P}_1'P_1Q_1^{-1}=\bar{Q}_1'^{-1}(\bar{Q}_1'Q_1-\bar{P}_1'P_1)Q_1^{-1} \\ &= \overline{(U_3P+U_4Q)}'^{-1}(\bar{Q}'Q-\bar{P}'P)(U_3P+U_4Q)^{-1} \\ &= \overline{(U_3Z+U_4)}'^{-1}(E-\bar{Z}Z)(U_3Z+U_4)^{-1} \end{aligned}$$

Taking the determinants, the lemma follows at once.

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1) Cf. H. Poincaré. Memoire sur les fonctions fuchsiennes. Memoire sur les fonctions zetafuchsinnes (Oeuvre Tom II.)

2) Masao Sugawara. Über eine allgemeine Theorie der fuchssche Gruppen und Theta-Reihen. Ann. Math. **41**; cited with S.