104. *Analytical Characterization of Displacements in General Poincaré Space.*

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In recent papers M. Sugawara has constructed a theory of automorphic functions of higher dimensions, as a generalization of Poincaré's theory[1]. He has considered the space $\mathfrak{A}_{(n)}$, whose points are symmetric matrices of order n with the property $E^{(n)}-\bar{Z}'Z>0$, and defined the displacements in $\mathfrak{A}_{(n)}$ as follows: Let $U=\begin{pmatrix}U_1U_2\\U_3U_4\end{pmatrix}$ be a matrix of order $2n$ satisfying the conditions $U'JU=J$, $U'S\bar{U}=S$, where $J=\begin{pmatrix}0 & E^{(n)}\\ -E^{(n)} & 0\end{pmatrix}$, $S=\begin{pmatrix}E^{(n)} & 0\\ 0 & -E^{(n)}\end{pmatrix}$. Then the transformation $W=(U_1Z+U_2)(U_3Z+U_4)^{-1}$ is called a displacement in $\mathfrak{A}_{(n)}$. In the classical case $n=1$, as is well known, the transformations of the type described above exhaust *all* the one-to-one analytic transformations which map $\mathfrak{A}_{(n)}$ into itsfelf. Then arises the problem: Does this fact remain true in our general case? In what follows this problem will be discussed for the spaces $\mathfrak{A}_{(n)}$ and $\mathfrak{A}_{(n,m)}$[2]. The answer is affirmative except for $\mathfrak{A}_{(n,n)}$. As in the classical case we are led to this result by an analogue to *Schwarz's lemma* in higher dimensions.

1. The set of all matrices of type (n, m) shall be denoted by $\mathfrak{R}_{(n,m)}$.

Theorem 1. If a mapping f of $\mathfrak{R}_{(n,m)}$ into itself satisfies the conditions: (1) $f(\alpha A+\beta B)=\alpha f(A)+\beta f(B)$, *($\alpha, \beta$ being complex numbers)* (2) *according as the rank of Z is 1 or 2, the rank of the image $f(Z)$ is 1 or $\geqq 2$, then the mapping f can be written in the following form: $f(Z)=AZB$, when $n\neq m$; $f(Z)=AZB$ or $AZ'B$, when $n=m$. Here A and B are non-singular constant matrices of orders n and m respectively.*

Proof. We shall denote the matrix units by $E_{\alpha\beta}$: the (α,β)-element of $E_{\alpha\beta}$ is equal to 1 and the other elements are all zeroes. For brevity let us call that a matrix A has the form (a) or (b), according as A can be written in the form $A=\sum_{\alpha=1}^{n}a_{\alpha1}E_{\alpha1}$ or $A=\sum_{\beta=1}^{m}a_{1\beta}E_{1\beta}$, where $a_{\alpha1}, a_{1\beta}$ are numbers. Now, by the condition (2), there exist non-singular matrices A_1 and B_1 (of orders n and m) such that $A_1f(E_{11})B_1=E_{11}$. Then $A_1f(E_{i1})B_1(i>1)$ has the form (a) or (b). For, if we put $A_1f(E_{i1})B_1=\sum_{\alpha,\beta}c_{\alpha\beta}E_{\alpha\beta}$ for a fixed i, we have, by the condition (2), $c_{11}c_{\alpha\beta}-c_{\alpha1}c_{1\beta}=0$

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1) M. Sugawara, Über eine allgemeine Theorie der Fuchsschen Gruppen und Theta-Reihen, Ann. Math., **41** (1940), 488–494. On the general zetafuchsian functions, Proc. **16** (1940), 367–372. A generalization of Poincaré-space, Proc. **16** (1940), 373–377, to be cited as $[S-3]$.

2) K. Morita, A remark on the theory of general fuchsian groups, Proc. **17** (1941), 233–237, to be cited as $[M-1]$.