102. *On Vector Lattice with a Unit, II.*

By Kôsaku YOSIDA and Masanori FUKAMIYA.
Mathematical Institute, Osaka Imperial University.

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§ 1. *Introduction and the theorems.* In a preceding note[1] one of the authors gave a representation of the vector lattice with a unit to obtain an algebraic proof of Kakutani-Krein's lattice-theoretic characterisation[2] of the space of continuous functions on a bicompact Hausdorff space. The purpose of the present note is to extend the result and to show that there exists a close analogy between the structures of the vector lattice and the algebras as in the case of the normed ring and the algebras[3].

A vector lattice E is a partially ordered real linear space, some of whose elements f are "non-negative" (written $f \geqq 0$) and in which[4]

(V 1): If $f \geqq 0$ and $\alpha \geqq 0$, then $\alpha f \geqq 0$.

(V 2): If $f \geqq 0$ and $-f \geqq 0$, then $f=0$.

(V 3): If $f \geqq 0$ and $g \geqq 0$, then $f+g \geqq 0$.

(V 4): E is a lattice by the semi-order relation $f \geqq g$ $(f-g \geqq 0)$.

In this note we further assume the existence of a "unit" $I>0$ satisfying

(V 5): For any $f \in E$ there exists $\alpha>0$ such that $-\alpha I \leqq f \leqq \alpha I$.

An element $f \in E$ is called "nilpotent" if $n|f|<I (n=1, 2, \dots)$. The set R of all the nilpotent elements f is called the "radical" of E. Surely R constitutes a linear subspace of E. Moreover it is easy to see that R is an "ideal" of E, viz. $f \in R$ and $|g| \leqq |f|$ imply $g \in R$. Here we put as usual $|f|=f^+-f^-$, $f^+=f \vee 0=\sup(f, 0)$, $f^-=f \wedge 0=\inf(f, 0)$.

Let N be a linear subspace of E. Then the linear congruence $a \equiv b$ (mod. N) is also a lattice-congruence:

$$a \equiv b,\ a' \equiv b' \pmod{N} \text{ implies } ab \equiv a'b' \pmod{N},$$

if and only if N is an ideal of E[5]. An ideal N is called "non-trivial" if $N \neq 0, E$. A non-trivial ideal N is called "maximal" if it is contained in no other ideal $\neq E$. Denote by $\mathfrak{N}$ the set of all the maximal ideals N of E. The residual class E/N of E mod. any ideal $N \in \mathfrak{N}$ is "simple", viz. E/N does not contain non-trivial ideals. It is proved

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1) K. Yosida: Proc. **17** (1941), 121–124. Cf. also M. H. Stone: Proc. Nat. Acad. Sci. **27** (1941), 83–87, and H. Nakano: Proc. **17** (1941), 311–317.

2) S. Kakutani: Proc. **16** (1940), 63–67. M. and S. Krein: C. R. URSS, **27** (1940), 427–430.

3) I. Gelfand: Rec. Math. **9** (1941), 1–24. We here express our hearty thanks to Tadasi Nakayama for his discussions during the preparation of the present note. He also obtained another proof of the theorem 1 below by considering the embedding of "lattice-groups" in a direct product of linearly ordered lattice-groups. See his paper shortly to appear in these Proceedings.

4) Small roman letters and small greek letters respectively denote elements $\in E$ and real numbers. We write $f>0$ if $f \geqq 0$ and $f \neq 0$.

5) Garrett Birkhoff: Lattice Theory, New York (1940), 109.