30. On some Property of Regular Functions in $|z|<1$.

By Tatsujiro SHIMIZU.

Mathematical Institute, Osaka Imperial University.

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§ 1. We shall introduce some of the directional maximum modulus of a regular function in the circle $|z|<1$, and give some theorem on it.

Let $f(z)$ be a regular function in $|z|<1$ and $M_\theta(r, \varepsilon)= \underset{|z|=r,\ \theta-\varepsilon<\operatorname{Arg} z<\theta+\varepsilon}{\text{l. u. b.}} |f(z)|$, ε being a positive number, and

$$\overline{\lim_{r\to1}} \frac{M_\theta(r, \varepsilon)}{\varphi(r)} = \overline{M}_\theta(1, \varepsilon)_\varphi$$

$$\underline{\lim_{r\to1}} \frac{M_\theta(r, \varepsilon)}{\varphi(r)} = \underline{M}_\theta(1, \varepsilon)_\varphi$$

where $\varphi(r)$ is a monotonously increasing function for $r\to1$.

Now
$$\underset{0<\varepsilon<\delta}{\text{g. l. b.}}\ \overline{M}_\theta(1, \varepsilon)_\varphi = \overline{M}_\theta(1)_\varphi\ {}^{1)}$$

$$\underset{0<\varepsilon<\delta}{\text{l. u. b.}}\ \underline{M}_\theta(1, \varepsilon)_\varphi = \underline{M}_\theta(1)_\varphi\,.$$

These measures are of some use for a regular function in $|z|<1$. In the following we shall consider the case $\varphi(r)\equiv1$ and denote by $\overline{M}_\theta(1)$ and $\underline{M}_\theta(1)$ respectively.

§ 2. Let E_θ be a set of θ, which is everywhere dense in $(0, 2\pi)$ and if $f(z)$ converges (to limits, ∞ included) for all θ, belonging to E_θ when $z=re^{i\theta}\to1$, θ being fixed, then we shall call $f(z)$ *has F-property.*

Let E_θ be a set of θ, which is everywhere dense in $(0, 2\pi)$ and if $\overline{M}_\theta(1)=\infty$ for all θ, belonging to E_θ, then we shall call $f(z)$ *has M-property.*

Theorem: Let $f(z)$ be regular in $|z|<1$ and have F- and M-properties, then the Riemann surface of the inverse function of $f(z)$ has no parts of boundary in the finite plane[2].

By to have parts of boundary[3], having α, β as the end-points, in the finite plane, we shall mean the following:

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1) l. u. b.=least upper bound.
g. l. b.=greatest lower bound.

2) A sort of modular functions has F- and M-properties. M-property is equivalent to the unboundness of $|f(z)|$ in any sector.

3) The boundary of the domain within the angle $<\alpha p\beta$ may be a line of singularity or a set of limit points of branch points. We suppose here α and β both lie in the finite plane.