3. *Positive Definite Integral Quadratic Forms and Generalized Potentials.*

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(Comm. by S. KAKEYA, M.I.A., Jan. 12, 1944.)

I. Positive definite integral quadratic forms.

Let Ω be a separable, metric space with metric $\rho(p, q)$ ($p \in \Omega$ and $q \in \Omega$). Suppose that the function $\Phi(p, q)$ defined for all points $p \in \Omega$ and $q \in \Omega$ satisfies the following conditions 1°)–5°):

1°) $\Phi(p, q) = \Phi(q, p) \geqq 0, \quad \Phi(p, p) = +\infty,$

2°) $\lim\limits_{\rho(p, q) \to 0} \Phi(p, q) = +\infty,$

3°) $\Phi(p, q)$ is a continuous function of (p, q) whenever $p \neq q$.

Before the condition 4°) is mentioned, it seems convenient to begin with some preliminary remarks.

Given a bounded Borel set E in Ω, let σ be any completely additive function of Borel sets on E. Then by Jordan's decomposition theorem[1], we may write $\sigma = \sigma^+ - \sigma^-$, where σ^+ and σ^- are the positive and negative variations of σ respectively, each of which is itself a non-negative, completely additive set-function defined for all Borel sets contained in E.

Now, consider the following integral:

$$\iint \Phi(p, q) d\sigma(p) d\tau(q)$$

$$= \lim_{N \to \infty} \iint \Phi_N(p, q) d\sigma^+(p) d\tau^+(q) + \lim_{N \to \infty} \iint \Phi_N(p, q) d\sigma^-(p) d\tau^-(q)$$
$$- \lim_{N \to \infty} \iint \Phi_N(p, q) d\sigma^-(p) d\tau^+(q) - \lim_{N \to \infty} \iint \Phi_N(p, q) d\sigma^+(p) d\tau^-(q),$$

where $\Phi_N(p, q) = \text{Min}\, \{N, \Phi(p, q)\}$.

$\int$ is used for $\int_E$ throughout this Note, so that $\iint$ for $\int_E\int_E$.

If all the four terms involved are finite, then the integral is said to be *absolutely convergent*. Thus the 4th condition is:

4°) $$+\infty \geqq \iint \Phi(p, q) d\sigma(p) d\sigma(q) \geqq 0$$

except when the integral is meaningless,

5°) if $\iint \Phi(p, \sigma) d\sigma(p) d\sigma(q) = 0$, then we have $\sigma(e) = 0$ for any Borel set $e \subset E$.

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1) S. Saks: Theory of the Integral, (1937), Chap. I.