2. *On Conformal Mapping of an Infinitely Multiply Connected Domain.*

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1. Let G be a Fuchsian group of linear transformations, which make $|z|<1$ invariant and D_0 be its fundamental domain containing $z=0$ and bounded by orthogonal circles to $|z|=1$ and D_n be its equivalent and e_n be the set on $|z|=1$, which belongs to the boundary of D_n. Let z_0 be a point in D_0 and z_n be its equivalent in D_n.

Theorem I. *If* $me_0>0$, *then* $\sum_{n=0}^{\infty} me_n=2\pi$ *and* $\sum_{n=0}^{\infty}(1-|z_n|)<0$.

$$\text{If } me_0=0, \text{ then} \sum_{n=0}^{\infty} me_n=0 \text{ and } \sum_{n=0}^{\infty}(1-|z_n|)=\infty ,$$

$$\sum_{n=0}^{\infty}(1-|z_n|)^2<\infty .$$

Let D be a domain on the w-plane, bounded by a closed set E, which contains at least three points and $\mathfrak{F}^{(\infty)}$ be the simply connected universal covering Riemann surface of the outside of E. We map $\mathfrak{F}^{(\infty)}$ on $|z|<1$ by $w=\varphi(z)$. R. Nevanlinna[1] proved that if cap. $E>0$, then E corresponds to a set of measure 2π on $|z|=1$ and if cap. $E=0$, then E corresponds to a set of measure zero on $|z|=1$, when z tends to $|z|=1$ non-tangentially. $\varphi(z)$ is automorphic with respect to a group G of linear transformations, which make $|z|<1$ invariant. Let D_0 be its fundamental domain containing $z=0$ and bounded by orthogonal circles to $|z|=1$ and D_n be its equivalent and e_n be the set on $|z|=1$, which belongs to the boundary of D_n. Then from Theorem I, we have easily:

Theorem II (*Precised form of R. Nevanlinna's theorem*).

$$\text{If cap. } E>0, \text{ then} \sum_{n=0}^{\infty} me_n=2\pi .$$

$$\text{If cap. } E=0, \text{ then} \sum_{n=0}^{\infty} me_n=0$$

2. Let F be a Riemann surface spread over the w-plane and $F^{(\infty)}$ be its covering Riemann surface of planar character and $\mathfrak{F}^{(\infty)}$ be its simply connected universal covering Riemann surface. We map $F^{(\infty)}$ on a schlicht domain D on the z-plane. D is the outside of a certain closed set E. We suppose that we can map $\mathfrak{F}^{(\infty)}$ on a unit circle $|\zeta|<1$ by $w=\varphi(\zeta)$. $\varphi(\zeta)$ is automorphic with respect to a group G of linear transformations, which make $|\zeta|<1$ invariant. Let D_0 be its fundamental domain containing $\zeta=0$ and bounded by orthogonal

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1) R. Nevanlinna: Eindeutige analytische Funktionen. Berlin, 1936.